Universidade de Aveiro Departamento de Engenharia Civil
2013

PIOTR SPUŚ

COST ANALYSIS OF REINFORCED CONCRETE SLABS AND COLUMNS

ANÁLISE DE CUSTOS DE LAJES E PILARES EM BETÃO ARMADO

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Dissertation was presented at University of Aveiro to fulfill the requirements for the degree of Master in Civil Engineering, held under the scientific guidance of Professor Miguel Morais, Assistant Professor, Department of Civil Engineering, University of Aveiro.

Dedykuję tę pracę mamie, w podzięce za lata poświęceń.

jury

president

Prof. Doutor Carlos Daniel Borges Coelho
assistant professor, University o Aveiro

Dr hab. Dariusz Heim, prof. TUL
associate professor, Lodz University of Technology

Prof. Doutor Miguel Nuno Lobato de Sousa Monteiro de Morais
assistant professor, University o Aveiro

agradecimentos

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keywords

rainforced concrete, design, safety check, ultimate limit states, cost analysis, flat slabs, reinforced concrete columns

abstract

The construction industry is increasingly looking for solutions that are both simple and effective and that provide cost savings, speed and flexibility of execution. Two-way slabs are a form of construction unique to reinforced concrete comparing with the other major structural materials. It is an efficient, economical, and widely used structural system.

The present dissertation aims to analyze and compare costs between four types of slabs: waffle slab with recuperate molds, flat slabs with drop panels, two-way slabs with beams and flat plates.

In this analysis the loads considered for the floors were of a residential type. The most common spans for slabs were considered.

For the analysis of the slabs the simplified methods were used. For the design, security checks and construction rules, it was considered the current legislation applied in the member countries of the European Committee for Standardization, namely the Eurocodes.

In order to compare the cost of usage of these four types of floor systems, in the analysis of the results it is shown the price for the necessary resources and the total cost of each slab for each study model per m^2 of total area of a building.

From this dissertation, the conclusion may be drawn that waffle slabs have a lower cost than flat slabs with enlarged column heads for all spans considered and respectively flat plates have a lower cost than slabs with beams. From all of the slabs, waffle slab is the most economical one in the range of considered spans.

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LIST OF SYMBOLS

Latin upper case letters

A_c	-	cross sectional area of concrete
A_s	-	cross sectional area of reinforcement
$A_{s,min}$	-	minimum cross sectional area of reinforcement
$A_{s,max}$	-	maximum cross sectional area of reinforcement
A_{sw}	-	cross sectional area of shear reinforcement
$A_{sw.min}$	-	maximum cross sectional area of shear reinforcement
$E_{c,eff}$	-	effective modulus of elasticity of concrete
EC2	-	Eurocode 2 (2010)
E_{cm}	-	secant modulus of elasticity of concrete
Es	-	design value of modulus of elasticity of reinforcing steel
Gk	-	characteristic permanent action
I	-	Second moment of area of concrete section
M_0	-	Reference moment
M_{Ed}	-	Design value of the applied internal bending moment
N_{Ed}	-	Design value of the applied axial force
Q_k	-	Characteristic variable action
SLS	-	Serviceability limit state
ULS	-	Ultimate limit state
V_{Ed}	-	Design value of the applied shear force
$V_{Rd,c}$	-	design shear resistance of the member without shear reinforcement

V_{Rd} - shear resistance of a member with shear reinforcement

Latin lower case letters

b - overall width of a cross-section

b_e - effective width of a cross-section

b_t - mean width of a cross-section

c_{min} - minimum concrete cover

$c_{min,b}$ - minimum cover due to bond requirement

$c_{min,dur}$ - minimum cover due to environmental conditions

c_{nom} - nominal concrete cover

d - effective modulus of elasticity of concrete

d_g - maximum size of aggregate

e - eccentricity

f_{cd} - design value of concrete compressive strength

f_{ck} - characteristic compressive cylinder strength of concrete at 28 days

f_{ctm} - mean value of axial tensile strength of concrete

f_{yd} - design yield strength of reinforcement

f_{yk} - characteristic yield strength of reinforcement

f_{ywd} - design yield of shear reinforcement

k - coefficient

h - height

$l_{b,rqd}$ - basic required anchorage length

l_{bd} - design anchorage length,

l_H	-	distance from face of a column to face of drop panel
s	-	spacing of stirrups
s_r	-	spacing of shear links in the radial direction
s_t	-	spacing of shear links in the tangential direction
u	-	
u_1	-	basic control perimeter
u_{1*}	-	reduced control perimeter
u_i	-	considered perimeter
$u_{out,ef}$	-	perimeter where shear reinforcement is no longer required
v_{Ed}	-	design value of the applied shear stress
$v_{Rd,c}$	-	design value of the punching shear resistance of a slab without punching shear reinforcement along the control section considered
$v_{Rd,max}$	-	design value of the maximum punching shear resistance along the control section considered
z	-	inner lever arm

Greek letters

ζ	-	reduction factor/distribution coefficient
Ø	-	diameter of a reinforcing bar
α	-	angle; coefficient
γ_c	-	partial factor for concrete
γ_s	-	partial factor for steel
δ	-	increment/redistribution ratio
μ	-	reduced moment
ν	-	Poisson's ratio

ρ_1	-	reinforcement ratio for longitudinal reinforcement
ρ_w	-	reinforcement ratio for shear reinforcement

1. INTRODUCTION

The construction industry is constantly looking for more effective solution. Especially in the era of worldwide crisis the approach to economic side of construction is significant. The construction of a slab because of its dimensions and area and form is very important part of the total cost of the building. The main aim of the dissertation is to compare reinforced concrete flooring systems. The comparison focuses on the economic appraisal (evaluation). Simplicity of realization which means lower labour costs and quantity of used materials are two crucial factors when considering economy of construction.

The type of construction is reinforced concrete construction. Method of construction is cast in place. A model of buildings with 6 storeys is under focus of this dissertation. This is the residential building without any special loads. This assumption helps to focus on the essence of considered problem. The plan of storeys of the building is simple and symmetrical. The common spans form 5.6m to 8.8m are taken into consideration. In the dissertation solid slabs spans in two directions with loads distributed uniformly were presented. The angles of considered slabs are right.

1.1. Objectives

The present work has as the main objective to make a comparative analysis of costs between different types of flat slabs and columns supporting them, in order to facilitate the decision of choosing the right slab among many types of flooring systems.

2. TWO-WAY SLAB SYSTEMS

Two-way concrete slabs are classified by load transfer system. The correct thickness is designed on the basis of an economical reinforcement which can be estimated as reinforcement ratio of $0,3 \div 0,9\%$ (Starosolski, 2003). Two-way slabs bend under load in both principal directions therefore there is a necessity to design layers of bars that are perpendicular to each other. Calculating a two-way slab uses the assumption that concrete is isotropic material. In restrained slabs the corners are prevented from lifting. In two-way slabs bending and torsion moments and shear forces are present. The characteristic dead load and imposed loads are approximately the same on the considered and adjacent panels.

2.1. Types of slabs

2.1.1. Flat plates

Flat plates are reinforced concrete slabs of uniform thickness that transfer loads directly to the supporting columns. Flat plate is a popular and well thought of slab mainly for its ease to arrangement the interior. Due to their simple formwork and reinforcing bar arrangement flat plates are quick to construct. They assure the highest level of flexibility in the arrangement of columns and partition walls and need the smallest storey height to provide headspace requirements. Flat plates have high fire resistance due to lack of sharp corners in which the phenomenon of spalling of the concrete may occur. Usually, the spandrel beams at the edges are designed. According to MacGregor (2012) for spans longer than 6.0m, the thickness required for the shear transfer of vertical loads to the columns exceeds that required for flexural strength. As a result, the concrete at the middle of the panel is not used efficiently. In this dissertation a confirmation of this statement is sought.

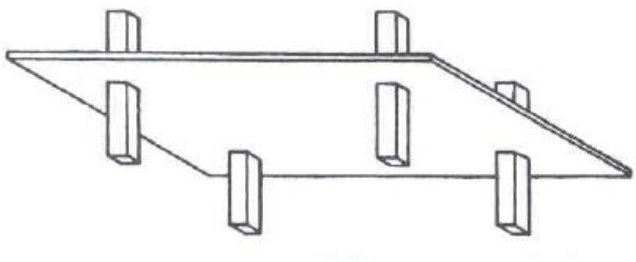


Figure 1: Example of a flat plate.

A possible problem in transferring the shear force may occur at the perimeter of the columns. It may therefore be necessary to increase column sizes or slab thickness or to use shearheads. Shearheads consist of steel I profile placed in the slab height over the column. Albeit using steel beams may seem expensive, it is still profitable when taking into consideration the simple and cheap formwork of such slab. For heavy industrial loads or long spans flat plates are not economical but they are commonly used in residential type of buildings.

2.1.2. Slabs with beams

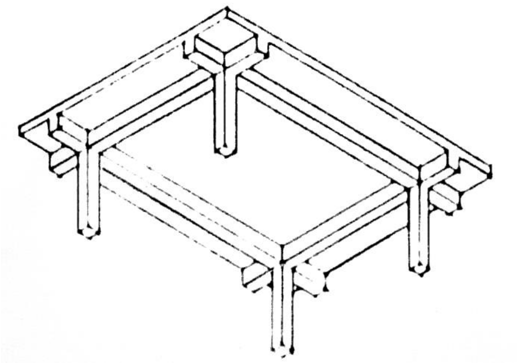


Figure 2: Two-way slab with beams.

Two-way slab with beams is used when the loads or spans or both are large and it is more economical to construct such a slab, despite the higher formwork expenditure. Putting it into other words, beams between columns strengthen the slab.

2.1.3. Waffle slabs

Waffle slabs (Figure 3) are constructed by arranging square molds with tapered sides with spaces between them. The concrete is later cast over and between the molds creating a characteristic waffle shape. These slabs are usually constructed solid near columns because there may be a problem with shearing force as in flat plates. Waffle slabs are rather thick and as a result they provide large moment arms for the reinforcing bars. Thanks to molds the weight of the concrete is significantly reduced without substantially changing the moment resistance. The span among ribs is typically less than 1,5m. Note that, near the columns, the full depth is retained for shear transfer of loads from the slab to the columns. This type of slab is also known as a two-way joist system. Waffle slabs are used for spans from 7.5m to 12.0m.

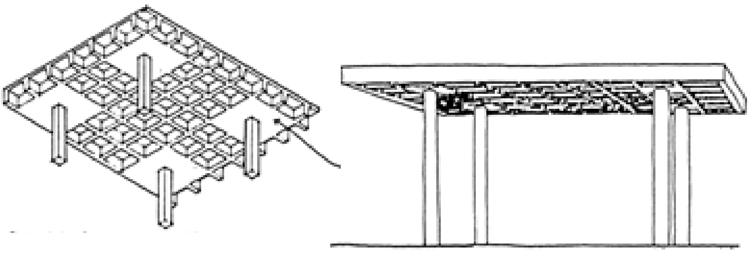


Figure 3: Examples of waffle slabs.

Solid heads in waffle slabs

Solid heads in waffle slabs are thicker areas of slab in proximity of columns. Their function is to transmit loads to columns and be the support for ribs and resist shear punching.

The height of solid heads is usually the same as the ribs. However, when large concentrations of loads occurs in the area of columns, thickness of solid heads may be greater. The half of a solid head dimension must be at least 0.15 times the span corresponding measured from the axis to the edge of the column (Figure 4). However, these are often constrained by the geometry of the molds which leads to the larger sizes required (Tesoro, 1991).

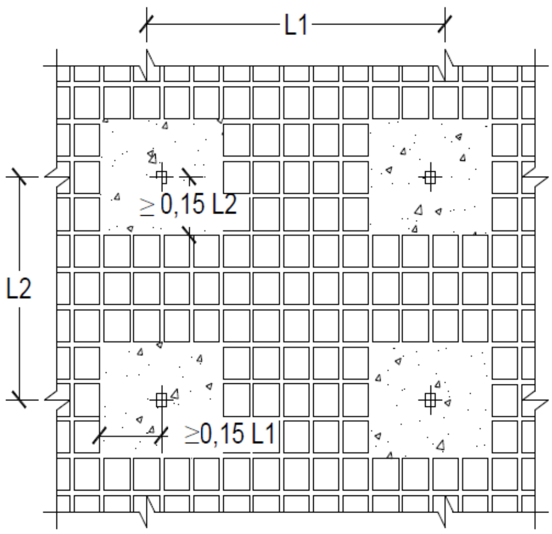


Figure 4: Dimensions of the solid head (Tesoro, 1991).

2.1.4. Flat slab

Flat slab (Figure 5) are reinforced concrete slabs with capitals, drop panels, or both. Although the framework is more expensive than for flat plates, the usage of concrete and steel is economical when heavy loads and long spans occur. They are particularly economical for structures where exposed drop panels or column capitals are acceptable. Flat slabs are used for spans from 6.0m to 9.0m.

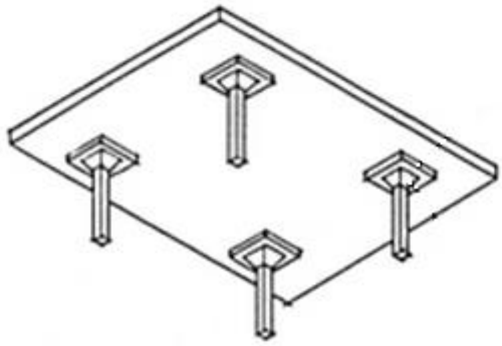


Figure 5: Flat slab with enlarged column heads and drop panels.

Column heads

The column head is the enlargement of the upper section of the column or thicker area of the slab. Currently, its scope has been reduced to industrial type buildings and commercial spaces, due to high overloads that these types of constructions lead (Tesoro, 1991). These elements are designed to resist moments and shear forces in the proximity of columns.

Drop panels

The drop panel stiffens the slab in the region of highest moments hence reduces the deflection. The presence of drop panel reduces the amount of negative-moment flexural reinforcement because effective depth of slab is increased. With the additional slab depth at the column, the area of the critical shear perimeter is increased. Minimum thickness of slab may be reduced by 10% if drop panels are present according to ACI Code Section 13.2.5. The thickness of the drop panel below the slab used in the calculations shall not be taken greater than one-fourth of the distance from the edge of the drop panel to the face of the column or column capital. If the drop panel were deeper than this, it is assumed that the maximum compression stresses would not flow down to the bottom of the drop panel, and thus, the full depth would not be effective, according to ACI Code Section 13.2.7. For economy in form construction, the thickness of the drop, shown as h_d in Figure 6 should be related to the actual timber dimensions plus thickness of plywood used for forms.

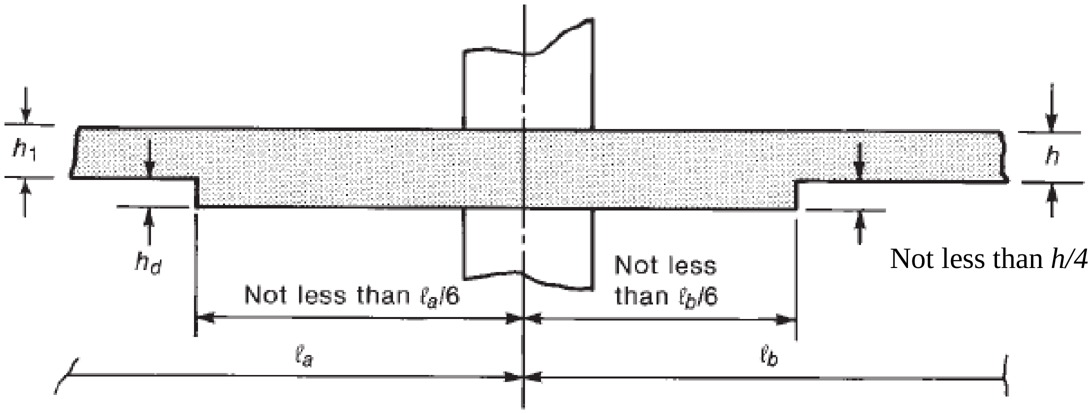


Figure 6: Minimum size of drop panel according to ACI Section 13.2.5.

2.2. Slab thickness

The parameters that determine the thickness of flat slabs are usually the punching shear and deformations.

According to Jiménez Montoya (2001) the thickness of flat slab should not be less than the minimum value of 12cm or 1/32 of the largest span. With respect to waffle slab, the thickness should not be less than the minimum of 15cm or 1/28 of the larger span.

In practice, these minimum values are not recommended, since the lead to deformation problems. The usual minimum thicknesses for flat slabs is 15cm or l/30 and for waffle slabs is 20 cm or l/25 respectively (Jiménez Montoya, 2001).

In order to control deformations Eurocode 2 (2010) in point *7.4.2* limits the span/effective height ratio. According to Eurocode 2 (2010), in the case of flat slabs with span greater than 8.5m, the span/effective height ratio should be multiplied by $8.5/l_{eff}$ (l_{eff} in meters). l_{eff} is defined in section 5.3.2.2 (1) of that standard. Regarding the compression area in the case of waffle slab, Eurocode 2 (2010) requires a thickness of topping slab exceeding 1/10 of the clear span between the ribs or 50mm.

3. CALCULATION OF RC SLABS AND COLUMNS

For the analysis of the slabs the simplified methods was used. For the design, security checks and construction rules, it was considered the current legislation applied in the member countries of the European Committee for Standardization, namely the Eurocodes. In order to make a decision among the use of specific type of slab, in the analysis of the results the price for the necessary resources and the total cost of each slab and columns for each study model should be delivered.

Not until deflection surpasses the obligatory standard, which is unacceptable, the value of a building on the real estate market is not affected. Therefore complying with the European standards in the matter of serviceability limit state is for every slab prerequisite.

3.1.Columns

To minimalize cost the columns have different cross section area on every two storeys. The cross section area of columns was calculated in accordance to EC2. When predimensioning of columns β coefficient which estimates the occurrence of eccentric support reaction with regard to control perimeter was taken into consideration. In Figure 6.21N recommended values of β are given. Values of β differ depend on the position of column in a structure.

3.2. Recommendation for torsion (BS 8110)

Torsion reinforcement should be provided on a basis of Table 1. It should consist of bars parallel to the edges of the slab and having at least 1/5 of the length of the shorter span. Both the top and bottom reinforcement are required.

Table 1: Percentage of area of reinforcement required for the mid-span design moment.

Position of a corner		both edges simply supported	one edge simply supported and other restrained	both edges restrained
Percentage of area required	[%]	75	50	0

To estimate bending moments in slab British Standard gives a designer tool of coefficients. In Table 2 the values for considered situations are shown. In the British Standard there are given conditions for the use of equations (1.1) and (1.2), that characteristic dead load and characteristic imposed loads are approximately the same on adjacent panels and the panel being considered. According to BS 8110 in slabs where corners are prevented from lifting, and provision for torsion is made, the maximum design moments per unit width are given by following equations:

$$m_{sx} = \beta_{sx} n l_x^2 \tag{3.1}$$

$$m_{sy} = \beta_{sy} n l_y^2 \tag{3.2}$$

where:

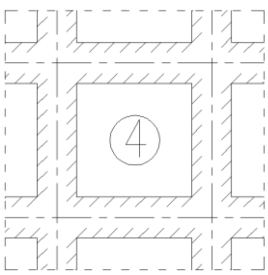
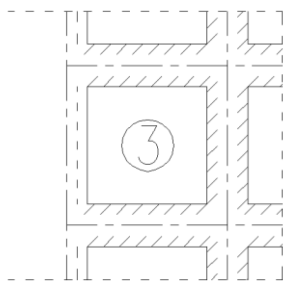
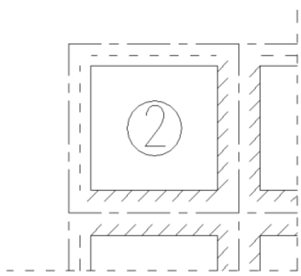
β_{sx}, β_{sy} – moment coefficients

n – total design ultimate load per unit area

l_x, l_y – length of shorter and longer side respectively

The span of adjacent panels is approximately equal the span of the considered panel in the direction perpendicular to the line of the common support.

Table 2: Bending moment coefficients for square panels supported on four sides with provision for torsion at corners for $l_y / l_x = 1.0$.

Interior panels		One edge discontinuous		Two adjacent edges discontinuous	
					
(1)	(2)	(1)	(2)	(1)	(2)
0.031	0.024	0.039	0.030	0.047	0.036

(1) Negative moment at continuous edge

(2) Positive moment at mid-span

When pre-dimensioning slab following equation is used:

$$\mu_{cs} = \frac{M_{Ed}}{bd^2 f_{cd}} \tag{3.3}$$

To estimate the effective depth of a cross-section d the $\mu_{cs} = 0.125$. The previous equation is transformed into:

$$d = \sqrt{\frac{M_{Ed}}{0.125 b f_{cd}}} \tag{3.4}$$

3.3. Beams

Beam are present in slabs with beams and as edge beams of other types of slabs. When considering a flat slab there is a question of proportion between beams and columns. The lower storey is analyzed the bigger columns are required to resist vertical force. It demands less labour to design beams which have the same width as the columns. However, on lower storeys the columns width is bigger than one of beams. Figure 7 show this case.

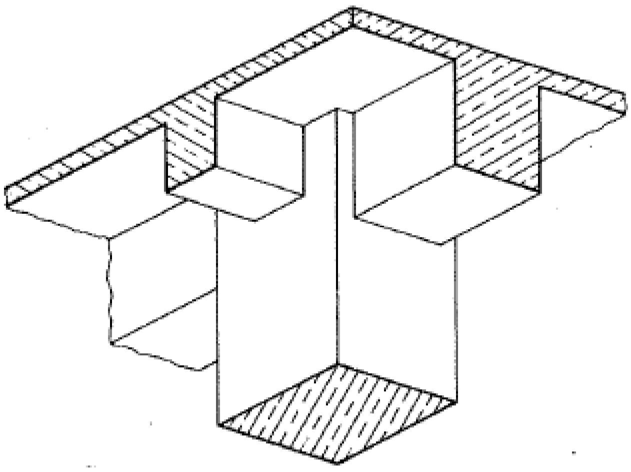


Figure 7: Not desired but practical way of designing beam-column connection.

When pre-dimensioning of beams following equation is used. To estimate the effective depth of a cross-section d the $\mu_{cs} = 0.3$.

$$d = \sqrt{\frac{M_{Ed}}{0.3 b f_{cd}}} \tag{3.5}$$

For the calculation of flat slabs there are several methods of analysis. When it comes to irregular complex slabs, commercial programs based on the finite element method should be used. They provide ease of use, fast and effective results of the calculation.
However, for situations when slabs are regular sometimes more effective mode is to use simplified methods, which are also easy and quick to use. Taking into account regularity of considered slabs simplified methods were used for the calculation internal forces and moments.

3.4. Equivalent frame analysis

According to Appendix I of Eurocode 2 equivalent frame analysis method consists in decomposing the structure in each of the orthogonal directions – longitudinal and transverse into frames consisting of columns and slab sections ranging between the center lines of adjacent panels. Panel is an area bounded by four adjacent supports. The slab can be analyzed using the methods applicable flat or plane frames. The calculation of the stiffness of the elements may be realized using their gross cross-sections. In the case of vertical loads, the rigidity takes into account the total width of the panels, whereas for horizontal loads must be considered 40% of this value to characterize a greater flexibility of the connections between the pillars and slabs structures flat slabs, when compared with the column/beam joints.

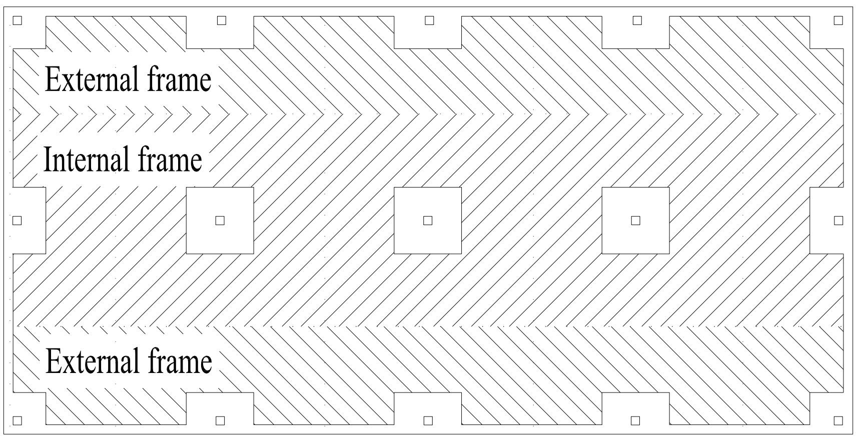


Figure 8: Division of frames for equivalent frame analysis.

The total load acting in each direction should be analyzed. Thus, every column gets two bending moments (M_x and M_y) and two axial forces are obtained for each of the orthogonal directions. For the design of the columns should be considered the two bending moments and the maximum value of the axial force obtained from the two directions (Tesoro, 1991).
In every considered frame bending moments distribute themselves within the limits recommended by Eurocode 2 (2010) in accordance with Table 3.
When in slab, drops are wider than one third of span, the column strips may be taken to be the width of drops. The width of middle strips should then be adjusted accordingly.

Table 3: Simplified apportionment of bending moment for a flat slab in Eurocode 2 (2010).

	Negative moments [%]	Positive moments [%]
Column strip	60 – 80 (80)	50 – 70 (60)
Middle strip	20 – 40 (20)	30 – 50 (40)
Note: used value in brackets		

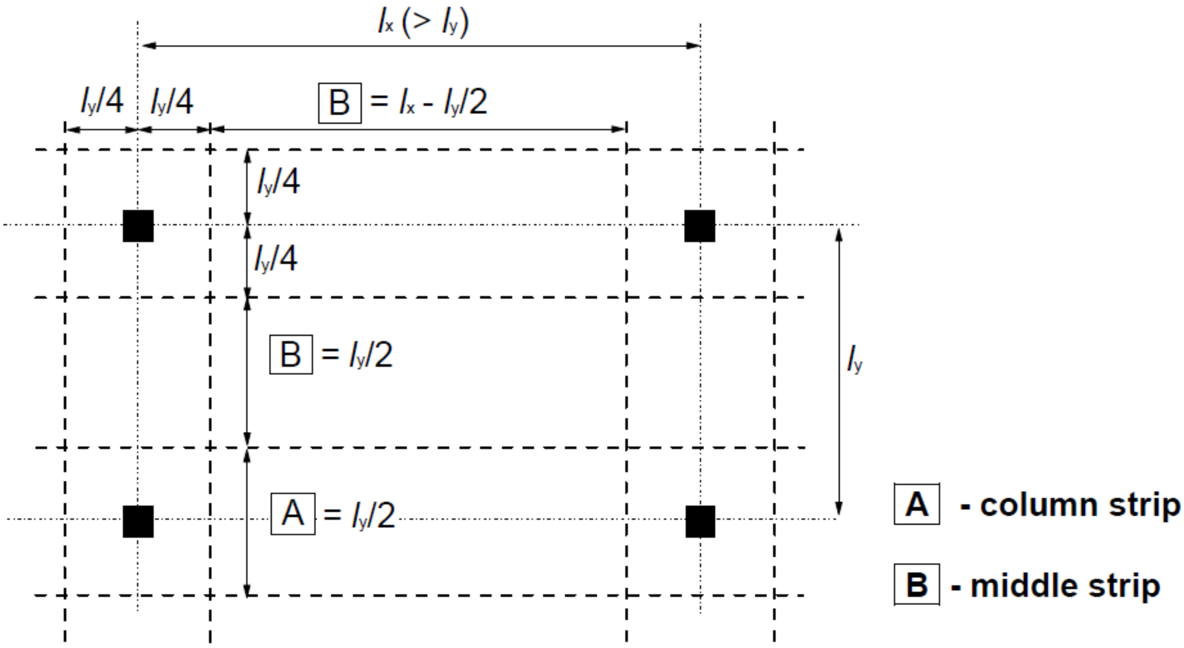


Figure 9: Division of panels in flat slabs (Eurocode 2, 2010).

If there are not perimeter beams, which are adequately designed for torsion, Eurocode limits moments transferred to edge or corner columns to the moment of resistance of a rectangular section equal to:

$$M = 0.17 b_e d^2 f_{ck} \tag{3.6}$$

b_e – effective width (Figure 10)

d – effective height

f_{ck} – characteristic value of strength of concrete

According to the Eurocode 2 (2010), the positive moments in the end span should be adjusted.

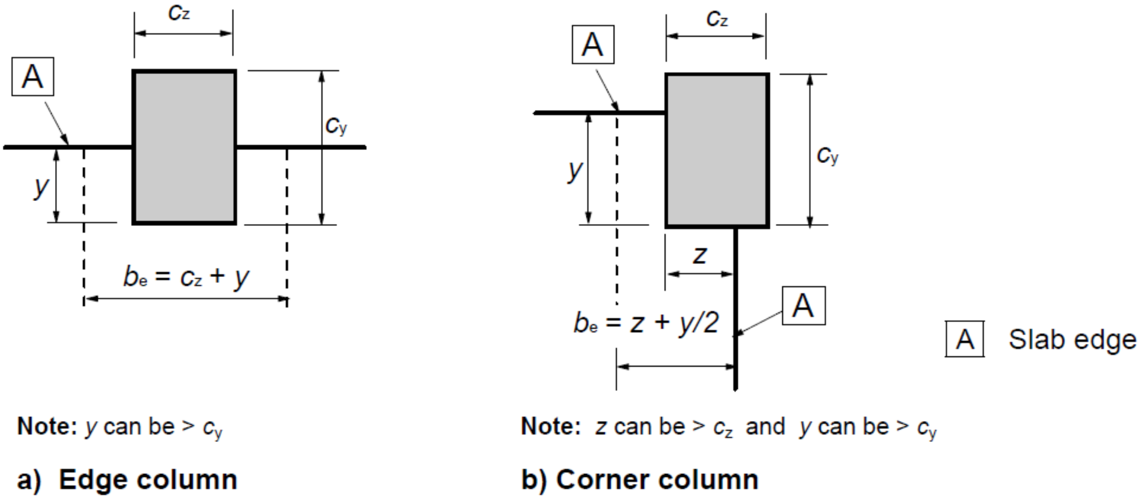
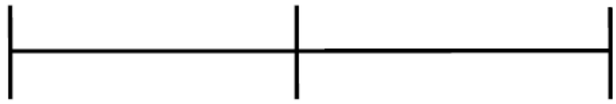


Figure 10: Effective width - b_e (Eurocode 2, 2010).

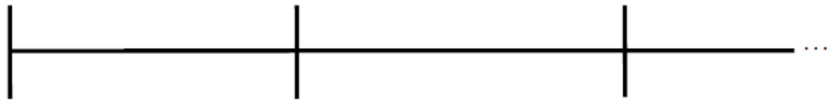
Cachim (2005) suggests the values listed in Table 4, which were derived from the regulation American standard ACI318-77:1983 and the British standard BS8110:1997. These values are valid under the following conditions:

- The length of the spans should not differ by more than 20%
- Actions should be predominantly distributed
- The variable load should be less than twice the amount of permanent load
- There is not redistribution of moments' values

Table 4: k values for calculating rough estimate of internal forces and moments (Cachim, 2005).



V		0,50		0,60		0,60		0,50
M	(2)	24	14		9		14	24



V		0,50		0,60		0,55		0,55		0,55	
M	(2)	24	14		10		15		11		15

On the supports considered an average of the spans

(2) extreme span with continuity, support beam

Given the *k* values presented in

, the bending moments and shear forces can be determined by the following expressions:

$$M = \frac{pl^2}{k} \tag{3.7}$$

$$V = kpl \tag{3.8}$$

From the mentioned simplified method, there is not possible to determine the shear force in slab or lighter efforts in prefabricated fascia. Taking this into account other criteria were introduce for obtaining these values.

For the determination of shear force in each rib of waffle slab (Figure 11) can be considered the following criteria in accordance with Teroso (1991):

$$Q_a = \frac{2}{9} KBLP_1 \tag{3.9}$$

L – span

B – width of equivalent panel

P_1 – total load per m^2

K – factor which takes into account the extreme moments (Figure 12)

The shear force which the beams situated outside the solid slab around column should resist is equal to:

$$V_{Ed} = \frac{Q_a}{n/2} \tag{3.10}$$

n – number of ribs in panel

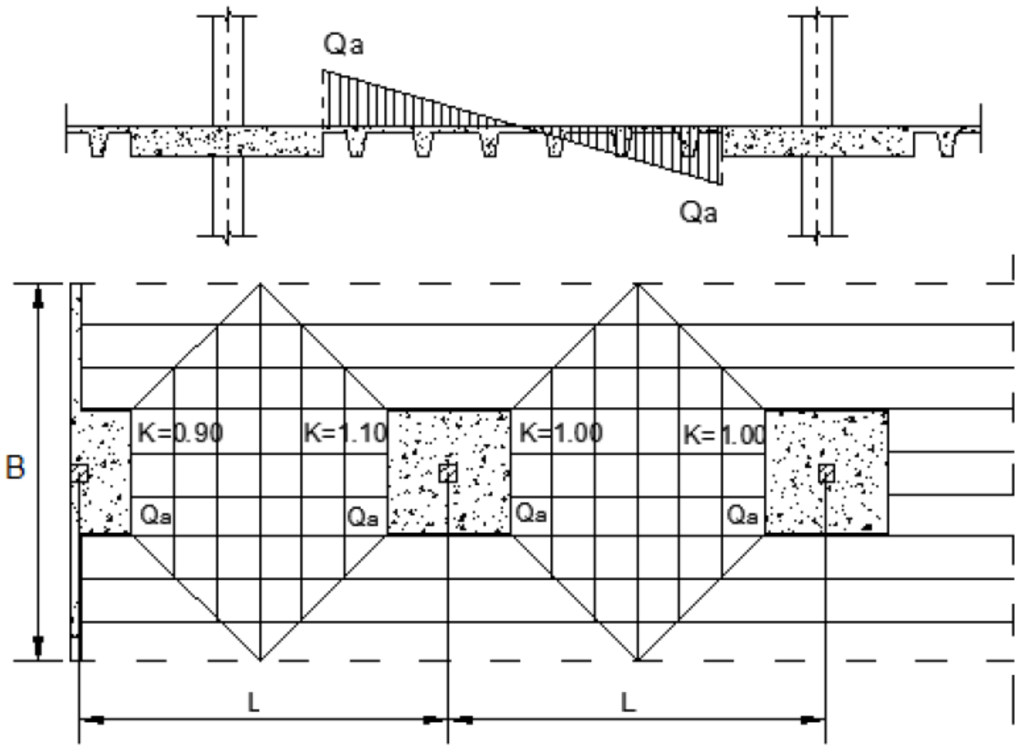


Figure 11: Coefficients to calculate shear force in beams (Tesoro, 1991).

For the simplified calculation of bending moments in edge beams, according to Tesoro (1991), the following expressions may be adopted:

$$M_0 = \frac{P_1 A L^2}{16} + \frac{P_2 L^2}{8} \tag{3.11}$$

$$M_I = 0.87\sigma K_1 M_0 \tag{3.12}$$

$$M_D = 0.87\sigma K_2 M_0 \tag{3.13}$$

$$M_V = 0.87\beta K_3 M_0 \tag{3.14}$$

L – span

B – width of equivalent panel

P_2 – linear load of external walls

K – coefficients illustrated in figure Figure 12

Values of σ and β are the percentage of positive and negative bending moments, taken from Table 5: . The coefficient that allows to take into consideration the size of the columns has the value of 0.87.

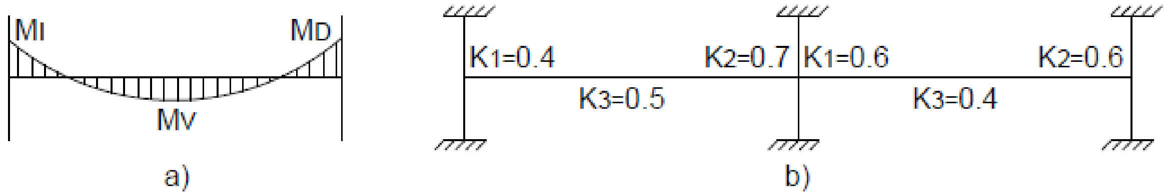


Figure 12: a) Coefficients to determine moments b) Coefficients K (Tesoro, 1991).

Table 5: Percentage of bending moments (Tesoro, 1991).

A [m]	M^- total σ [%]	M^+ total β [%]
6.0	38	32
6.5	33	28
7.0	32	27
>7.0	30	25

According to Tesoro (1991), the cross-section where the edge beam required to be checked for shear resistance is in proximity of solid heads. For the determination of shear the following practical criteria can be applied, since in practice these beams are symmetrically armed (Figure 13).

$$V_{Ed,1} = V_1\left(1 - \frac{1}{6y}\right)\sigma \quad if\ x > x \tag{3.15}$$

$$V_{Ed,1} = V_1\left(1 - \frac{1}{6x}\right)\sigma \quad if\ y < x \tag{3.16}$$

where σ is the percentage of the total shear to designate the virtual frame edge beam, which may be obtained from the same table as for moments (Table 5:).

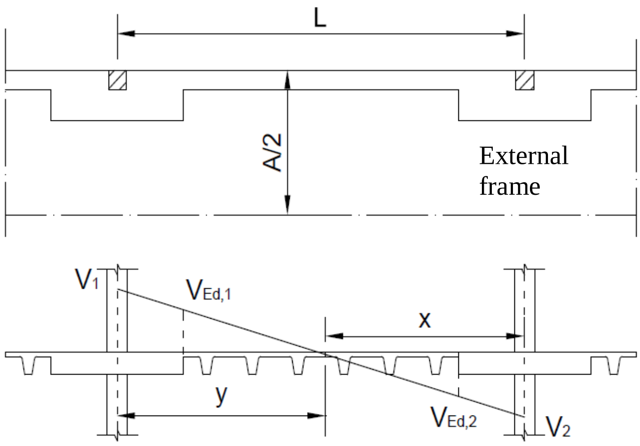


Figure 13: Shear force in edge beams (Tesoro, 1991).

3.5. General rules of design

3.5.1. Concrete cover

Regarding the cover to reinforcement, Eurocode 2 (2010), establishes the minimum cover $c_{\min}$, whose function is to ensure the effective transmission of the forces of adhesion, protection of steel against corrosion and adequate fire resistance, which is determined from the following expression:

$$c_{\min} = \max\left\{c_{\min,b}; c_{\min,dur} + \Delta c_{dur,\gamma} - \Delta c_{dur,st} - \Delta c_{dur,add}; 10mm\right\} \tag{3.17}$$

where:

$c_{\min,b}$ - minimum cover due to bond requirement,

$c_{\min,dur}$ - minimum cover due to environmental conditions,

$\Delta c_{dur,\gamma}$ - additive safety element

$\Delta c_{dur,st}$ - reduction of minimum cover for use of stainless steel

$\Delta c_{dur,add}$ - reduction of minimum cover for use of additional protection

According to Eurocode 2 the values of $\Delta c_{dur,\gamma}$, $\Delta c_{dur,st}$, $\Delta c_{dur,add}$ are equal $0mm$.

Given that the bars are arranged separately, the minimum value of the minimum concrete cover $c_{\min}$, is given by the diameter of the bar. As for the value of the minimum cover on the environmental conditions $c_{\min,dur}$, this can be taken from Table 4.4N of Eurocode 2 (2010), taking into account the structural class defined from Table 4.3N of the regulation and exposure class. The nominal cover c_{nom}, is determined by the sum of the minimum cover $c_{\min}$ and a margin for calculating tolerances execution Δc_{dev}. The value recommended by Eurocode 2 (2010), for this last parameter is 10 mm.

$$c_{nom} = c_{\min} + \Delta c_{dev} \tag{3.18}$$

3.5.2. Distance between bars

For the concrete to be placed and compacted satisfactorily for the development of adequate bond, the spacing of bars shall be as follows.

$$d = \max\{\varnothing; d_g + 5mm; 20mm\} \tag{3.19}$$

where:

Ø – diameter of a bar

d_g - dimension of aggregates

3.5.3. Anchorage of longitudinal reinforcement

To calculate the length of anchorage the following calculation sequence was took into account, according to Eurocode 2.

The design value of the ultimate bond stress is calculated as follows:

$$f_{bd} = 2.25\eta_1\eta_2 f_{ctd} \tag{3.20}$$

where:

f_{ctd} - the design value of concrete tensile strength

η_1 - coefficient related to the quality of the bond condition and the position of the bar during concreting. When ‘good’ conditions are obtained $\eta_1 = 1$ In other cases $\eta_2 = 0.7$

η_2 - coefficient related to the diameter of bars:

$\eta_2 = 1.0$ for bars with diameter equal or less than 32mm

$\eta_2 = (132 - \varnothing)/100$ for bars with diameter of more than 32mm

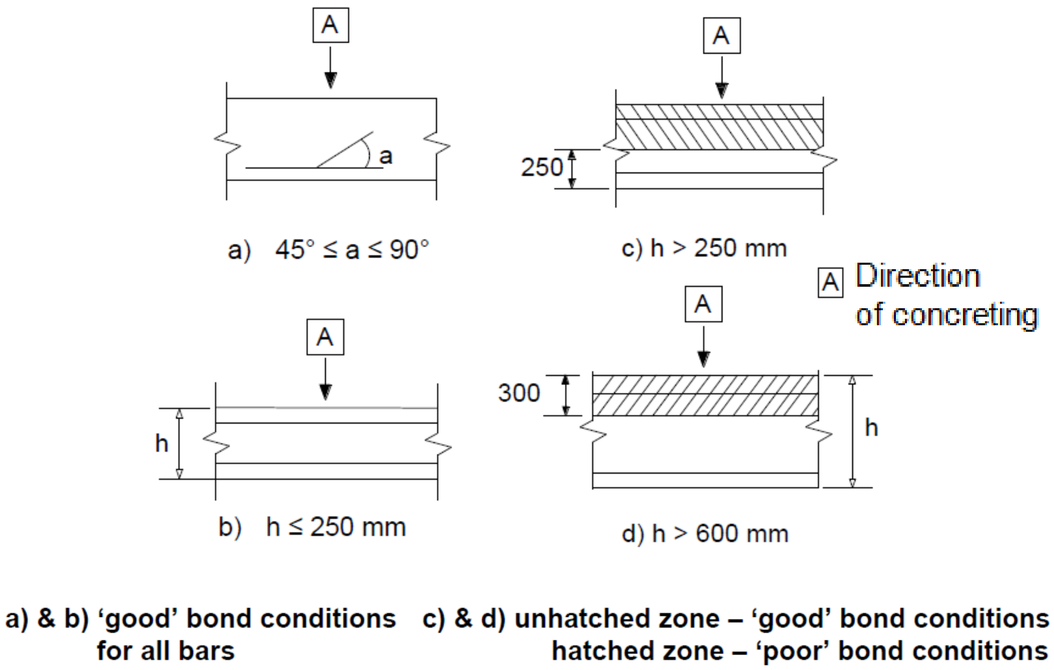


Figure 14: Description of bond conditions Eurocode 2 (2010).

Having regard to Figure 14, the quality of bond conditions were considered "good" in the case of the top reinforcement, when the thickness of the slab is below 250mm. Anchorage for the bottom reinforcement of the ribs were also considered as "good". The basic required anchorage length of a straight bar:

$$l_{b,rqd} = (\phi / 4)\ (\sigma_{sd} / f_{bd}) \tag{3.21}$$

where:

σ_{sd} - the design stress at the position from where the anchorage is measured from

l_{bd} - the design anchorage length:

$$l_{bd} = \alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 l_{b,rqd} \geq l_{b,\min} \tag{3.22}$$

Where coefficients $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are given in Table 8.2 of Eurocode 2 (2010). The minimum anchorage length for anchorages in tension is shown by the equation (1.19). The minimum anchorage length for anchorages in compression is shown by the equation (1.20).

$$l_{b,\min} > \max\{0{,}3 l_{b,rqd}; 10\phi; 100mm\} \tag{3.23}$$

$$l_{b,\min} > \max\{0{,}6 l_{b,rqd}; 10\phi; 100mm\} \tag{3.24}$$

3.5.4. Anchorage of shear reinforcement

A bar should be provided inside a hook or bend. Type of anchorage of shear reinforcement is shown in the Figure 15.

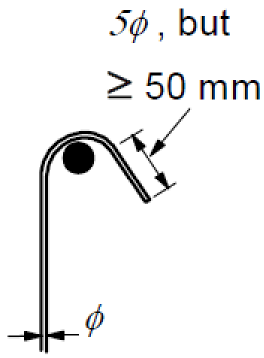


Figure 15: Anchorage of shear reinforcement (Eurocode 2, 2010).

3.6. Ultimate limit states

According to Eurocode 0 (2009), ultimate limit states refer to the safety of people and or safety of the structure. In light of this, the verifications of the ultimate limit states were made to bending, shear and punching.

3.6.1. Bending

Calculation of reinforcement

For the design of bending reinforcement simplified expressions were used for rectangular sections subjected to simple bending, based on the parabola-rectangle diagram for concrete.

$$\mu = \frac{M_{Ed}}{bd^2 f_{cd}} \tag{3.25}$$

$$\omega = 0.973(1 - \sqrt{1 - 2.056\mu}) \tag{3.26}$$

$$A_s = \frac{\omega b d f_{cd}}{f_{yd}} \tag{3.27}$$

where:

μ – coefficient

M_{Ed} – applied bending moment

b – width of cross-section

ω – coefficient

f_{cd} – design strength of concrete

f_{yd} – design strength of steel

d – effective height

For the calculation of the lower reinforcement of waffle slab, the ribs are regarded as T-beams, in which initially the calculated position of the neutral axis in order to verify that the height of the compression zone was lower than the height of the flange.
With regard to minimum and maximum area of bending reinforcement, Eurocode 2 (2010), recommends the value given by the following expression:

$$A_{s,\min} = m\acute{a}x\left\{0{,}26\frac{f_{ctm}}{f_{yk}}b_t d\,;0{,}0013 b_t d\right\} \tag{3.28}$$

$$A_{s,\max} = 0{,}04 A_c \tag{3.29}$$

where:

b_t – average width of bending zone
f_{ctm} – characteristic mean value of tensile strength
A_c – cross-section of concrete
f_{yk} – characteristic value of strength of steel

In zones of positive moments where crack control is required, minimum amount of bonded reinforcement is required to control cracking in areas where tension is expected. The required minimum areas of reinforcement may be calculated as follows. In profiled cross sections like T-beam, minimum reinforcement should be determined for the individual parts of section according to Eurocode 2 (2010).

$$A_{s,min}\sigma_s = k_c k f_{ct,eff} A_{ct} \tag{3.30}$$

where:

$A_{s,min}$ – minimum area of reinforcing steel within the tensile zone

A_{ct} – area of concrete within tensile zone

σ_s – absolute value of the maximum stress permitted in the reinforcement immediately after formation of the crack

$f_{ct,eff}$ – mean value of the tensile strength of the concrete effective at the time when the cracks may first be expected to occur

k - is the coefficient which allows for the effect of non-uniform self-equilibrating stresses, which lead to a reduction of restraint forces

k_c - is a coefficient which takes account of the stress distribution within the section immediately prior to cracking and of the change of the lever arm

Detailing of reinforcement

When detailing of reinforcement in flat slabs, in case of analyzing using simplified methods, principles shown in Figure 16 should be used.

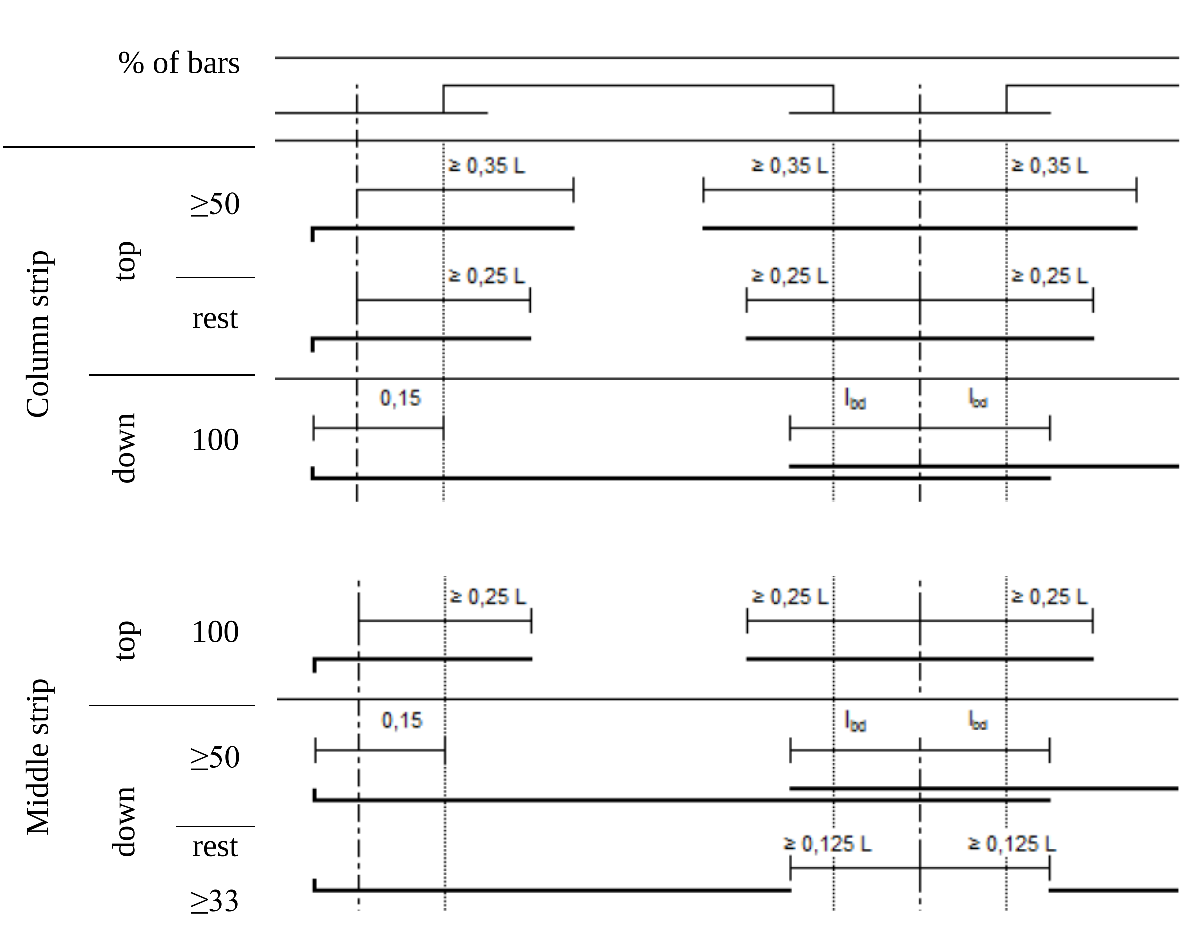


Figure 16: Minimum anchorage of reinforcement in flat slabs (Tesoro, 1991).

Minimum areas of reinforcement are given in order to prevent a brittle failure, wide cracks and also to resist forces arising from restrained actions (Eurocode 2, 2010).

3.6.2. Shear force

According to point 6.2.1 (4) of Eurocode 2 (2010) the minimum shear reinforcement may be omitted in members such as slabs. In waffle slabs verification of shear force in beams should be checked near solid capitals.

For verification of shear resistance according to Eurocode 2 (2010), the following methods should be provided:

- If $V_{Ed} \leq V_{Rd,c}$ there is no need to use reinforcement
- If $V_{Ed} > V_{Rd,c}$ there is a need to use reinforcement meeting condition $V_{Ed} \leq V_{Rd}$

where:

V_{Ed} - design value of the applied shear force

$V_{Rd,c}$ - design shear resistance of the member without shear reinforcement

V_{Rd} - shear resistance of a member with shear reinforcement

Design shear resistance of the member without shear reinforcement

The value of design resistance of the member without shear reinforcement may be calculated as follows:

$$V_{Rd,c} = max\left\{\left[\frac{0.18}{1.5}k(100\rho_l f_{ck})^{1/3} + k_1\sigma_{cp}\right]b_w d;\ \left(0.035k^{3/2}f_{ck}^{1/2} + k_1\sigma_{cp}\right)b_w d\right\} \tag{3.31}$$

where:

f_{ck} - is in *MPa*

$k = min\left\{1 + \sqrt{\frac{200}{d}}; 2.0\right\}$, d is in *mm*

$\rho_l = min\left\{\frac{A_{sl}}{b_w d}, 0.02\right\}$

A_{sl} - is the area of the tensile reinforcement, which extends $\geq (l_{bd} + d)$, beyond the section considered (Figure 17)

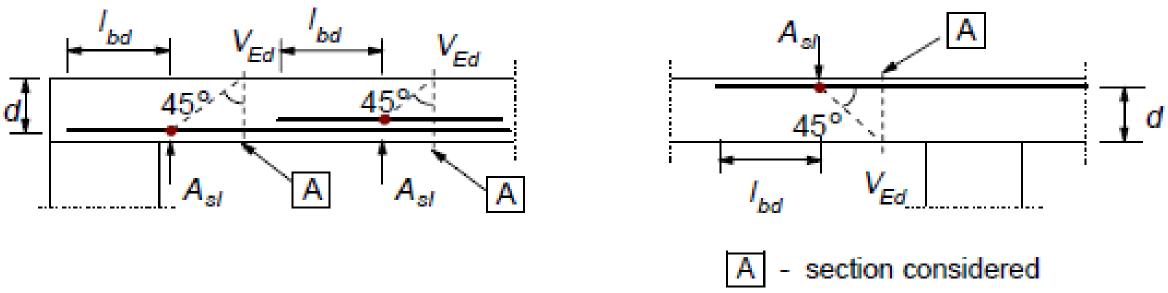


Figure 17: Definition of A_{sl} (Eurocode 2, 2010).

b_w - is the smallest width of the cross-section in the tensile area in *mm*

$\sigma_{cp} = min\left\{\frac{N_{Ed}}{A_c}; 0.2f_{cd}\right\}$ is in *MPa*

N_{Ed} - is the axial force in the cross-section due to loading or prestressing in *N*

A_c - is the area of concrete cross section in mm^2

Calculation of shear reinforcement

When calculate shear reinforcement Eurocode 2 (2010) gives the limiting values of $cot\theta$ between 1.0 and 2.5. For members with vertical shear reinforcement, the shear resistance, V_{Rd} is the smaller value of:

$$V_{Rd} = \min\left(\frac{\alpha_{cw} b_w z v_1 f_{cd}}{cot\theta + tan\theta}; \frac{A_{sw}}{s} z f_{wyd} cot\theta\right) \tag{3.32}$$

where:

α_{cw} – coefficient taking account of the state of the stress in the compression chord

b_w – minimum width between tension and compression chords

z – inner lever arm, the approximate value z = 0,9d may normally be used

v_1 - strength reduction factor for concrete cracked in shear, $v = 0.6\left[1 - \frac{f_{ck}}{250}\right]$

Taking this into account, we calculated the shear reinforcement from the expression:

$$\frac{A_{sw}}{s} \geq \frac{V_{Ed}}{z f_{ywd} cot\theta} \tag{3.33}$$

where:

A_{sw} – area of the transverse section of steel

s – spacing of stirrups

f_{ywd} - design yield strength of the shear reinforcement

Verification of following expression is required by Eurocode 2 (2010).

$$\frac{A_{sw}}{s} \geq \frac{0.08\sqrt{f_{ck}}}{f_{yk}} b_w sin\alpha \tag{3.34}$$

where:

A_{sw} – area of the transverse section of steel

s – spacing of stirrups

b_w – width of element

α – angle between shear reinforcement and longitudinal axis of the element (45° -90°)

In zones of the ribs of waffle slab where shear reinforcement is not required, the minimal reinforcement should be introduced.

3.6.3. Shear punching

Punching shear can result from a concentrated load or reaction acting on a relatively small area, called the loaded area A_{load} of a slab (Eurocode 2, 2010).This type of failure is dangerous, as manifested abruptly without prior notice. A place where slab has a contact which column is a crucial in every type of flat slab because in this place maximum moments and maximum shear force may be found. In waffle slabs a zone where slab is supported by a column is usually a massive part of the slab with the height of the ribs (Figure 19). It is recommended to avoid placement punching reinforcement in vicinity of the majority of columns (Tesoro, 1991). To reduce problems connected to punching shear, increased height of the slab can be used as well as increased size of the columns or designing column heads. In the Figure 18 cracking pattern of slab after shear punching failure was shown.

Figure 18: Cracking pattern of slab after failure.

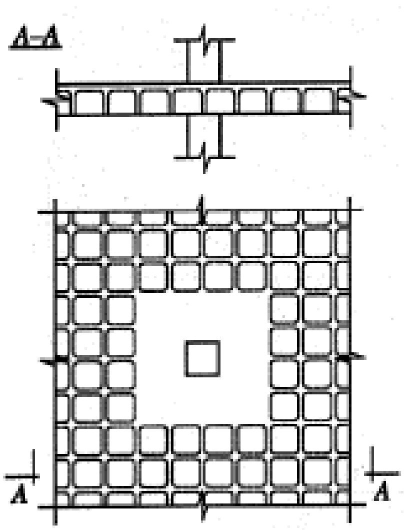


Figure 19: Example of support zone in waffle slabs (Starosolski, 2003).

According to Eurocode 2 (2010) the verification of punching resistance consists of checking the shear resistance at the face of the column and at the basic control perimeter u_1. The basic control perimeter u_1 may normally be taken to be at a distance of double effective depth from the face of column which is a loaded area in this case. If shear reinforcement is required a further perimeter u_{out} or $u_{out,ef}$ should be found where shear reinforcement is no longer required.

For verification of punching shear resistance according to Eurocode 2 (2010), the following checks should be provided:

- $v_{Ed} \leq v_{Rd,max}$ at the column perimeter
- If $v_{Ed} \leq v_{Rd,c}$ punching shear reinforcement is not necessary
- If $v_{Ed} > v_{Rd,c}$ there is a need to use reinforcement

where:

v_{Ed} - design value of the applied shear stress

$v_{Rd,c}$ - design value of the punching shear resistance of a slab without punching shear reinforcement along the control section considered

$v_{Rd,max}$ - design value of the maximum punching shear resistance along the control section considered

Design value of the applied shear stress v_{Ed}

Punching shear stress, when the support reaction is eccentric should be taken as:

$$v_{Ed} = \beta \frac{V_{Ed}}{u_i d} \tag{3.35}$$

where:

β – coefficient connected with eccentricity of support reaction

V_{Ed} – applied shear force

u_i – length of the control perimeter being considered

d - effective depth of the slab is taken as constant as follows:

$$d = d_{eff} = \frac{d_y + d_z}{2} \tag{3.36}$$

where:

d_y, d_z – effective depths of the reinforcement in two orthogonal directions

β coefficient in different cases

Internal columns

$$\beta = 1 + k\frac{M_{Ed}}{V_{Ed}} \cdot \frac{u_1}{W_1} \tag{3.37}$$

where:

k - coefficient dependent on the ratio between the column dimensionsc_1, c_2: its value is a function of the proportions of the unbalanced moment transmitted by uneven shear and by bending and torsion (see Table 6:)

Table 6: Values of k for rectangular loaded areas.

c_1/c_2	≤ 0.5	1.0	2.0	≥ 3.0
k	0.45	0.60	0.70	0.80

u_1 – length of the basic control perimeter

W_1 - corresponds to a distribution of shear and is a function of the basic control perimeter u_1

$$W_1 = \frac{c_1^2}{2} + c_1c_2 + 4c_2d + 16d^2 + 2\pi dc_1 \tag{3.38}$$

where:

c_1 - column dimension parallel to the eccentricity of the load

c_2 - column dimension perpendicular to the eccentricity of the load

For an internal rectangular column where the loading is eccentric to both axes, the following approximate expression for β may be used:

$$\beta = 1 + 1.8\sqrt{\left(\frac{e_y}{b_z}\right)^2 + \left(\frac{e_z}{b_y}\right)^2} \tag{3.39}$$

where:

e_y and e_z are the eccentricities M_{Ed}/V_{Ed} along *y* and *z* axes respectively

b_z and b_y is the dimensions of the control perimeter (see Figure 20)

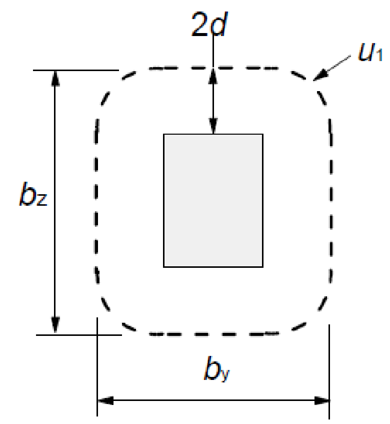


Figure 20: Basic control perimeter.

Edge columns

According to Eurocode 2 (2010) for edge column connections, where the eccentricity perpendicular to the slab edge (resulting from a moment about an axis parallel to the slab edge) is toward the interior and there is no eccentricity parallel to the edge, the punching force may be considered to be uniformly distributed along the control perimeter u_{1*} as shown in Figure 22.

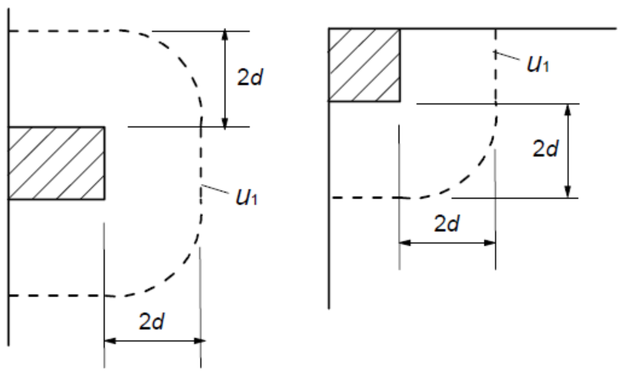


Figure 21: Basic control perimeters close to edge or corner (Eurocode 2, 2010).

Corner columns

According to Eurocode 2 (2010), for corner column connections, where the eccentricity is toward the interior of the slab, it is assumed that the punching force is uniformly distributed along the reduced control perimeter u_{1*}, as defined in Figure 22. The β-value may then be considered as:

$$\beta = \frac{u_1}{u_{1*}} \tag{3.40}$$

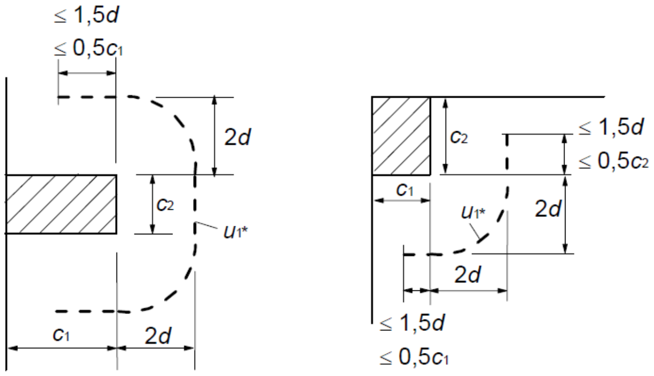


Figure 22: Reduced basic control perimeter u_{1*}

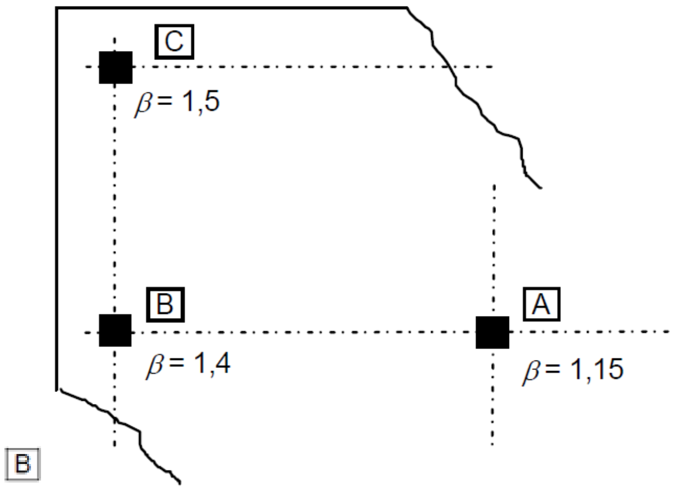


Figure 23: Coefficients recommended in eurocode 2, 2010.

The design punching shear resistance of slabs without shear reinforcement $v_{Rd,c}$

The design punching shear resistance of slabs without shear reinforcement may be calculated as follows:

$$v_{Rd,c} = max\left\{\frac{0.18}{1.5}k(100\rho_l f_{ck})^{1/3} + k_1\sigma_{cp}; 0.035k^{3/2}f_{ck}^{1/2} + k_1\sigma_{cp}\right\} \quad (3.41)$$

where:

f_{ck} - is in MPa

$k = min\left\{1 + \sqrt{\frac{200}{d}}; 2.0\right\}$, d is in mm

$\rho_l = min\{\sqrt{\rho_{ly}\rho_{lz}}, 0.02\}$

ρ_{ly}, ρ_{lz} - relate to the bonded tension steel in y- and z- directions respectively. They should be calculated as mean values taking into account a slab width equal to the column width plus 3*d* each side

b_w - is the smallest width of the cross-section in the tensile area in *mm*

$\sigma_{cp} = \frac{\sigma_{cy}+\sigma_{cz}}{2}$ is in *MPa*

σ_{cy}, σ_{cz} - normal concrete stresses in the critical section in y- and z- directions in *MPa*, positive if compression

Control perimeters

In the case when the column head dimensions meet condition $l_H < 2h_H$, verifying the shear punching stresses should be performed in the control perimeter outside the capital.

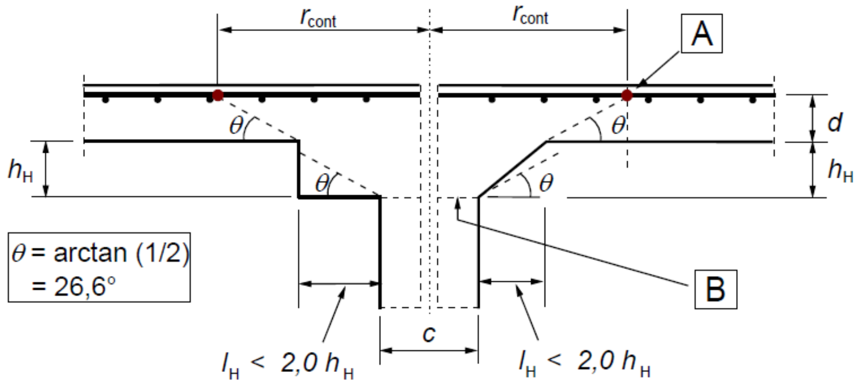


Figure 24: Slab with enlarged head where $l_H < 2h_H$ (Eurocode 2, 2010).

For the columns considered in this dissertation, which are quadratic, the value $r_{\ cont}$ may be taken as:

$$r_{\ cont} = 2d + 0.56(c + 2l_H) \tag{3.42}$$

where:

d - effective depth of the column head or slab

c – dimension of column side

l_H – see Figure 24

In the case when the column head dimensions meet condition $l_H > 2h_H$ (Figure 25), verifying the shear punching stresses should be performed in control sections both within the head and in the slab.

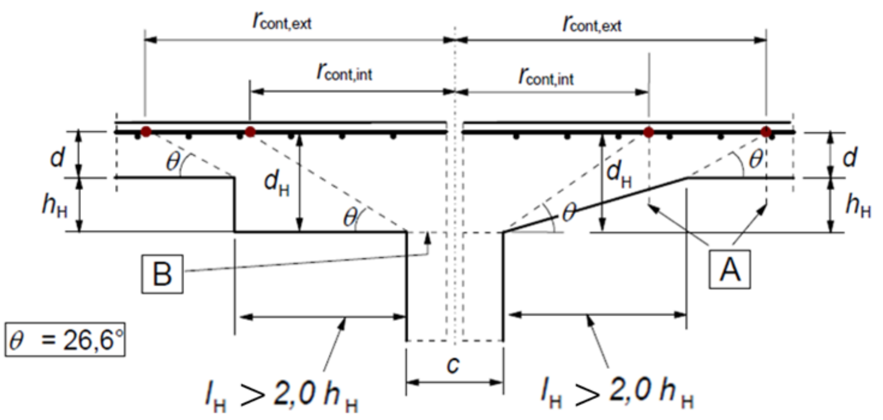


Figure 25: Slab with enlarged head where $l_H > 2h_H$ (Eurocode 2, 2010).

Calculation of punching shear reinforcement

To find a necessary shear reinforcement a following equation may be used:

$$\frac{A_{sw}}{s_r} = \frac{v_{Ed} - 0.75v_{Rd,c}}{1.5df_{ywd,ef}\frac{1}{u_1d}sin\alpha} \quad (3.43)$$

where:

A_{sw} - area of one perimeter of shear reinforcement around the column in *mm*^{2}

s_r - radial spacing of perimeters of shear reinforcement in *mm*

$f_{ywd,ef}$ - effective design strength of the punching shear reinforcement in *MPa*

$$f_{ywd,ef} = \max(250 + 0.25d;\ f_{ywd}) \quad (3.44)$$

α - angle between the shear reinforcement and the plane of the slab

d - effective depth of the slab taken as average between orthogonal directions

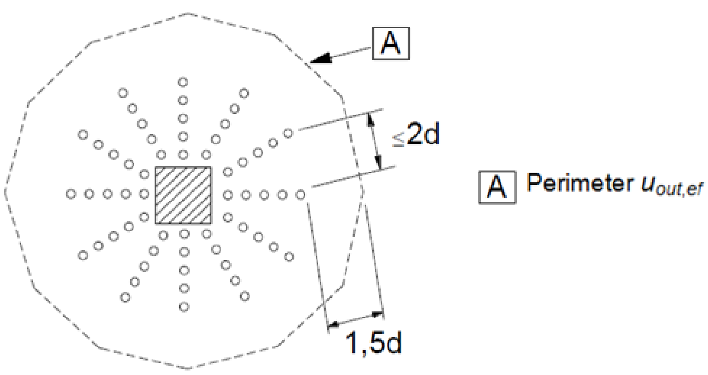


Figure 26: Control perimeter at internal column (Eurocode 2, 2010)

The outermost perimeter of shear reinforcement should be placed at a distance not greater than 1.5*d* within u_{out} according to Eurocode 2 (2010) (Figure 26).

$$u_{out} = \frac{\beta v_{Ed}}{v_{Rd,c} d} \qquad (3.45)$$

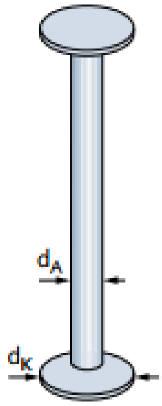


Figure 27: Stud considered to resist punching shear.

According to Eurocode 2 (2010), punching shear reinforcement should be located in harmony with Figure 28. Punching shear reinforcement must be constructed with at least two perimeters of shear reinforcement. The spacing of the link leg perimeters should not exceed *0.75d*. The distance between the face of a support and the nearest shear reinforcement taken into account in the design should not exceed d/2.

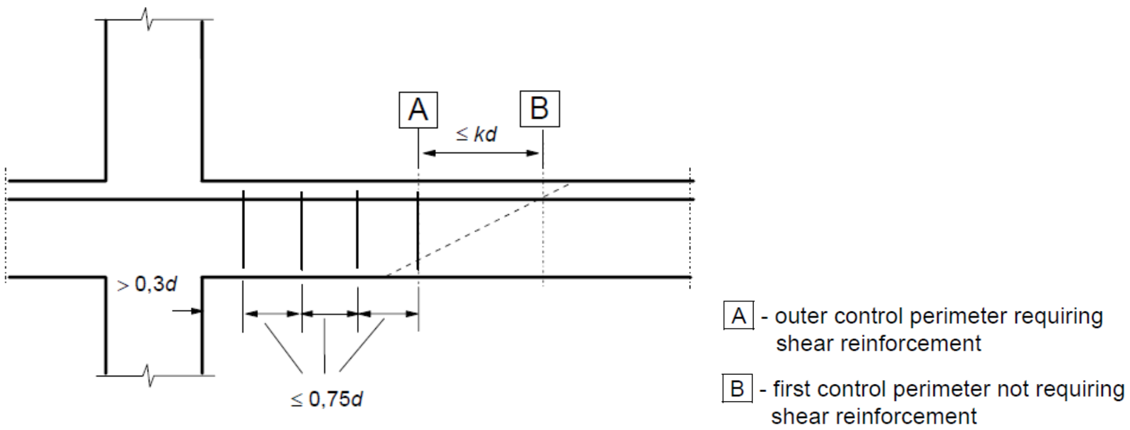


Figure 28: Spacing of links (Eurocode 2, 2010).

The area link leg should not be lesser than:

$$A_{sw,min} = \frac{1.5sin\alpha + cos\alpha}{s_r s_t} \geq 0.08\frac{\sqrt{f_{ck}}}{f_{yk}} \tag{3.46}$$

where:

s_r – spacing of shear links in the radial direction

s_t - spacing of shear links in the tangential direction

Punching shear resistance adjacent to the column.

$$v_{Ed} = \frac{\beta V_{Ed}}{u_0 d} \leq v_{Rd,max} \tag{3.47}$$

where:

u_0 –	for an interior column	u_0 = length of column periphery in mm
	for an edge column	$u_0 = c_2 + 3d \leq c_2 + 2c_1$ in mm
	for a corner column	$u_0 = 3d \leq c_2 + c_1$ in mm

$c_2 + c_1$ – column dimensions

$v_{Rd,max} = 0.5vf_{cd}$

$v = 0.6\left[1 - \frac{f_{ck}}{250}\right]$

3.7. Serviceability limit state

The height of slabs were assumed to avoid deflection problems in normal circumstances and as follows avoid necessity of calculation them. To avoid it the span/depth ratio of every type and span of considered slabs lies in the limits proposed by Eurocode 2 (2010).

Calculation of crack widths was not conducted.

4. MODELS OF STUDY

For comparison of the slabs simplified methods were conducted. To find bending moments occurring in slabs, method of Cachim (2005) where used. This method can be used for at least two spans of similar dimensions. The structure were divided into frames in both orthogonal directions as shown in Figure 30. The bending moment values repeat themselves in every internal frame and external frame respectively in both directions.

In the waffle slabs and slab with enlargement column heads, the dead load is not uniform along the span, as areas along the columns have an higher weight than the rest of the range. With this in mind, an average load along the span was calculated, in order to use the method described by Cachim (2005), for the approximate calculation of the internal forces.

4.1. Materials

In the project concrete of class C30/37 was used. Exposure class related to environmental conditions in accordance with EN 206-1, was chosen as XC1 – concrete inside buildings with low air humidity. Reinforcement was made from A500 NR SD steel.

4.2. Actions

To determine dimensions of structural members knowledge of occurring loads and their sources is inevitable. Actions taken into consideration were shown in Table 7: .
To represent floor and ceilings weight and HVAC systems without self-weight of structure the fallowing characteristic value of permanent actions was applied. Dead load is serious factor for calculating a slab. It is directly connected with thickness of slab which is calculated from Table 7.4N [EC1]. Presence of drop panels or column capitals increases dead load of the slab. In the waffle slab dead load is decreased by application of molds. A characteristic value of imposed loads of typical for residential dwellings was applied in accordance with Eurocode 1 (2009) for category A. Load from wind is not taken into consideration for being considered insubstantial for the structure. Snow does not have a great influence on the structure in Portuguese climate. Considerable majority of Portugal

has very low characteristic value of snow load of 0.1 kN/m^2 up to 0.3 kN/m^2 in minor areas.

Table 7: Considered actions

Permanent actions	Dead load (reinforced concrete)	25.0 kN/m^3
	Uniformly imposed	5.0 kN/m^2
	External wall	9.5 kN/m
Variable actions	Uniformly imposed	2,0 kN/m^2

4.3.Edge beams

Dimensions of edge beams differ depending on the dimensions of molds in waffle slab and for other types of flat slabs, the dimensions of edge and corner columns and thickness of the external wall it may be hidden in. A spandrel beam which is another name for edge beam is shown in Figure 29.

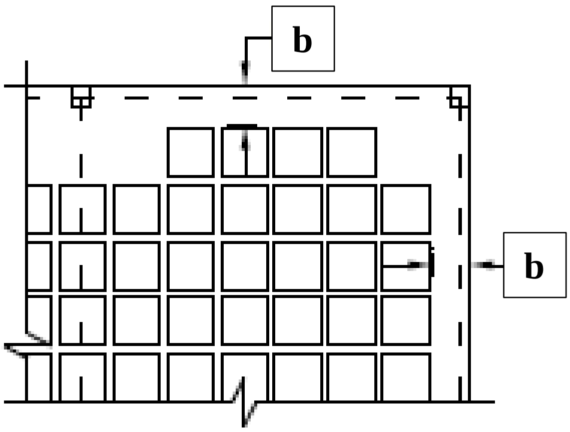


Figure 29: Edge beams in waffle slab.

4.4. Columns

In the considered model of a building different dimensions of columns are used because. It depends on the position of the column. The positions are as follow: internal, edge and corner. The dimensions of columns change every 2 storeys to obtain more economical values. The percentage of steel in the column, the reinforcement ratio, is taken as 2%. The mean value of steel weight for m^3 of concrete is assumed as 150kg. The dimensions of columns can be found in Appendix A.1 – A.4 and the cost of columns can be found in the Appendix C.

4.5. Cost of materials, formwork and labour

The prices used in the calculations of cost of resources are presented in Table 8. The prices are valid for construction industry in Portugal.

Table 8: Cost of materials

Concrete C30/37			
	price		
	[euro/m^3]		
	77.5		
Steel A500 NR SD			
	diameter	price	area
	[mm]	[euro/kg]	[cm^2]
	6	0.85	0.28
	8	0.83	0.50
	10	0.80	0.79
	12	0.78	1.13
	16	0.77	2.01
	20	0.77	3.14
	25	0.79	3.85

To verify what is the difference between applying two different solutions of reinforcement a simple model was considered. When building 100m of columns or beams with required area of reinforcement of 12cm^2 using 6 bars of diameter of 16mm instead of only 4 bars of diameter of 20mm we can safe 30.82€ on steel (Table 9:). But on the other hand fewer bars in the cross-section mean less labor and the same effects obtained faster and with the less workload for employees. Every day of construction, only in terms of renting unnecessary tools, costs significant amount of money. The amount of 30.82 euro seem to be very little sum of money in this case.

Table 9: Price comparison of reinforcement in 100m of beam

Quantity of bars in cross-section	Diameter of bar	Area of reinforcement	Weight of 1m	Price of steel in 100m beam
	[mm]	[cm^2]	[kg]	[euro]
4	20	12.56	9.88	760.76
6	16	12.06	9.48	729.96

5. ANALYSIS OF RESULTS

In this chapter the comparison among various types of flat slabs is presented. The waffle slabs and the slabs with column heads are considered in spans between *7.2m*, *8.0m* and *8.8m* The flat plates and the slabs with beams comparison is provided for spans between *5.6m* and *7.2m* also with the graduation of *0.8m*. The comparison is made to emerge the most economical solution for the range of spans considered.

The comparison of cost of columns for buildings with different slab systems is presented in this chapter. The cost is estimated per m^2 of total area of the building. This means that the area of ground floor and the area of six upper storeys were summed up to calculate the total area.

In Appendix A.1 a complete example of calculation of structural members for waffle slab with grid of columns with span of *7.2m*, applied forces and moments are presented,. The adequate calculation for the same grid of columns were made for the slab with column heads in Appendix A.2, the flat plate in Appendix A.3 and the slab with beams in Appendix A.4.

In Appendix B tables of costs for each slab of different systems were presented with division of materials, labour force and formwork.

In Appendix C tables of costs of columns for each slab of different systems were presented.

5.1. Internal forces and bending moments

For comparison of internal forces and bending moments in this chapter the representative model with a span of 7.2m among the grid of columns was chosen as the span, which each of analyzed types of slabs has.

For a better understanding of the following tables in the Figure 30 the division of a slab into frames was presented and in the Figure 31 plan of the slab with beams was presented with the considered cross-sections.

The divisions of the gateways and the tracks were made according to Figure 3.3 and Figure 3.4.

For comparison of the flat slabs simplified methods were conducted. These methods are not very precise compared to the finite element method, which leads to more realistic values, since it takes into account the variations of loads and allows to obtain a better estimation of deformation. The simplified method is precise enough to compare the slabs and obtain relevant conclusions.

To find bending moments occurring in slabs, method of Cachim (2005) was used. This method can be applied for at least two spans of similar dimensions. The structure was divided into frames in both orthogonal directions as shown in *Figure 30*. The values of bending moment repeat themselves in every internal frame and external frame respectively in both directions.

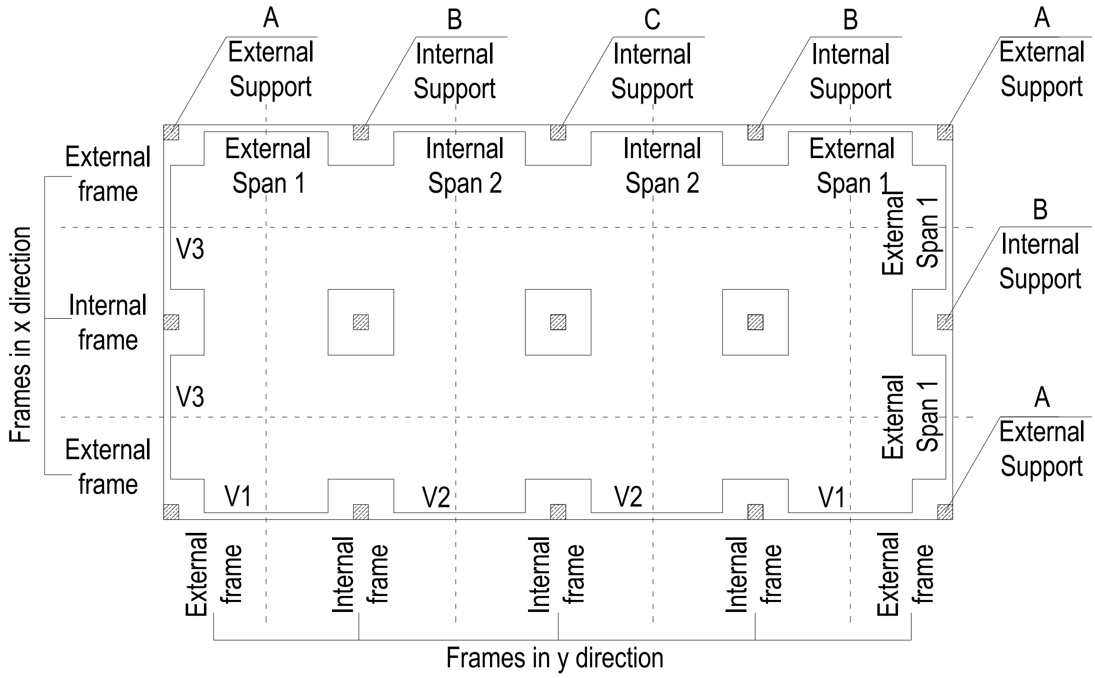


Figure 30: Division of slab into frames for the method of Cachim (2005).

The values in Table 10 and Table 11: were calculated using method of professor Cachim (2005). The conclusion may be made that the values of the bending moments among different types of slabs are similar to each other. This is due to the fact that the applied loads are similar. The difference in loading lays in difference among dead load of specific type of the slab. The simplified method of finding bending moments is slabs proposed by professor Cachim (2005) requires knowing the uniformly distributed load. To calculate the uniformly distributed load on every type of slabs the approximation of dead load value should be made. For example when calculating dead load of slab with column heads, there is the area of slab with normal height and the area of drop panels with bigger height and different rigidity but there is no mode to take it into consideration, the approximation should be done.

Table 10: Bending moments in considered slabs in direction x for span of 7.2m [kNm].

Type of slab	Strip	External support A	External span 1	Internal support B	Internal span 2	Internal support C
Slab with column heads	Column	188.66	277.22	452.79	258.74	411.62
	Middle	80.85	184.81	194.05	172.49	176.41
Waffle slab	Column	189.29	278.14	454.29	259.59	412.99
	Middle	81.12	185.42	194.70	173.06	177.00
Flat plate	Column	227.42	334.16	545.80	311.89	496.18
	Middle	97.46	222.78	233.91	207.92	212.65

Table 11: Bending moments in considered slabs in direction y for span of 7.2m [kNm].

Type of slab	Strip	External support A	External span 1	Internal support B
Slab with column heads	Column	188.66	277.22	503.10
	Middle	80.85	184.81	215.61
Waffle slab	Column	189.29	278.14	504.77
	Middle	81.12	185.42	216.33
Flat plate	Column	227.42	334.16	606.45
	Middle	97.46	222.78	259.91

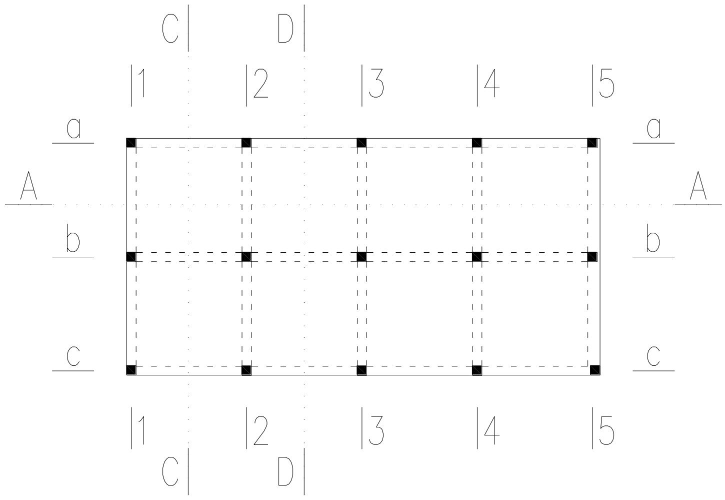


Figure 31: Plan of a slab with beams.

The slabs with beams were designed using British standard *BS 8110:1997*. For the different types of supports there are determined coefficients to calculate the bending moments. The plan of a considered slab with beams is shown in Figure 31. Table 12: and Table 13: show the values of bending moments in the slab of span of *7.2m*.

Table 12: Bending moments in the considered slab with beams in direction x for span of 7.2m [kNm/m]

Cross section	$M_{Ed,1-1}$	$M_{Ed,1-2}$	$M_{Ed,2-2}$	$M_{Ed,2-3}$	$M_{Ed,3-3}$
	External support	External mid-span	Internal support	Internal mid-span	Internal support
A-A	0.00	38.28	-45.72	31.90	-41.47

Table 13: Bending moments in the considered slab with beams in direction y for span of 7.2m [kNm/m]

Cross section	$M_{Ed,a-a}$	$M_{Ed,a-b}$	$M_{Ed,b-b}$
	External support	External mid-span	Internal support
C-C	0.00	38.28	-45.72
D-D	0.00	31.90	-41.47

In Table 14 axial forces transmitted from a single slab of different types to the columns in different position as internal, edge and corner are shown. The values are similar in every case because the applied load is the same.

Table 14: Axial force in columns from the slab of different types for span of 7.2m [kN]

Type of slab	Internal column	Edge column	Corner column
Slab with column heads	894.24	539.46	315.90
Waffle slab	901.37	543.02	317.68
Slab with beams	1222.72	744.27	398.71
Flat plate	1082.94	633.81	363.07

In Table 15: shear forces in the edge beams are presented. The values for waffle slab and slab with column heads are similar, and the values for the flat slab are bigger.

Table 15: Shear forces in the egde beams

Beam	Slab	Left support	Right support
V1	Flat plate	60.99	73.18
	Waffle slab	51.91	62.29
	Slab with column heads	51.76	62.11
V2	Flat plate	67.09	67.09
	Waffle slab	57.10	57.10
	Slab with column heads	56.94	56.94
V3	Flat plate	60.99	73.18
	Waffle slab	51.91	62.29
	Slab with column heads	51.76	62.11

In Table 16 bending moments in the edge beams are presented. The values for waffle slab and slab with column heads are similar, and the values for the flat slab are slightly bigger.

Table 16: Bending moments in the edge beams.

Beam	Slab	Left support	Mid-span	Right support
V1	Flat plate	31.87	33.19	55.76
	Waffle slab	27.60	28.75	48.30
	Slab with column heads	27.53	28.68	48.18
V2	Flat plate	47.80	26.55	47.80
	Waffle slab	41.40	23.00	41.40
	Slab with column heads	41.29	22.94	41.29
V3	Flat plate	31.87	33.19	55.76
	Waffle slab	27.60	28.75	48.30
	Slab with column heads	27.53	28.68	48.18

5.2. Comparison of costs

A comparative cost analysis performed in this thesis encompasses four types of flat slabs, waffle slab, slab with column heads, slab with beams and flat plate as well as the columns which are required to support these slabs.

5.2.1. Cost structure of slabs with column heads

The price of concrete required per m^2 for slabs with column heads of spans between *7.2m* and *8.8m* considered every *0.8m* are very similar. The cost of steel used in discussed slab grows with the span and reaches value of *27.23€/m*^{2} for span of *8.8m*. The top reinforcement of the slab with column heads was increased to satisfy the good practice of not using shear punching reinforcement in proximity of every column both within the column heads and in the slab. This solution helped to avoid placing studs or bended bars used as shear punching reinforcement in the proximity of internal columns but simultaneously increased the labour which must be spent for placing longitudinal reinforcement above the column heads in the top part of the slab. The formwork and labour stay at the same level for different spans and falls within the price ambit of 18.03*€/m*^{2} to 19.82*€/m*^{2}. The cost structure divided into different resources of those cost is presented in the Figure 32. The cost of slabs and columns supporting them may be seen in the Figure 33.

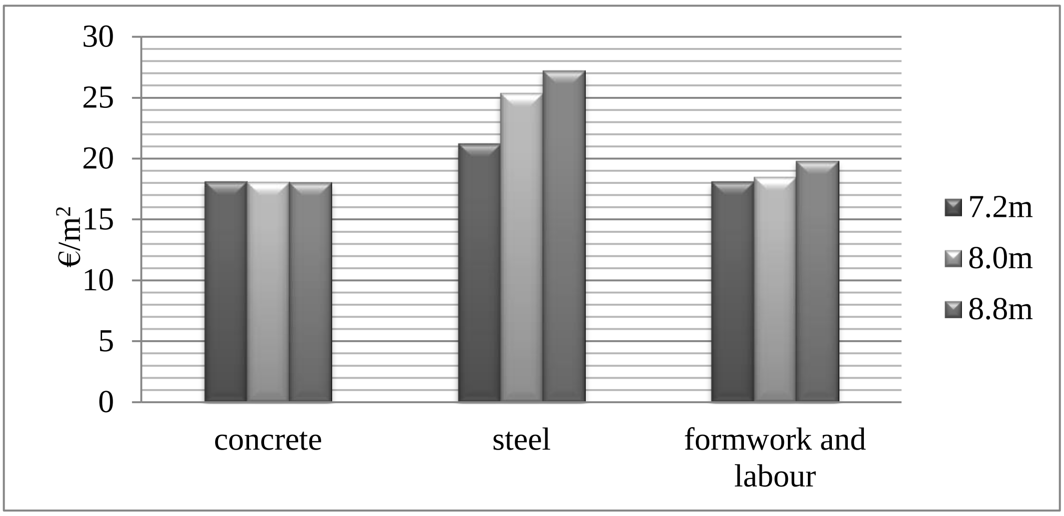


Figure 32: Comparison of costs of materials and formwork and labour per m^2 of total area of building for slabs with column heads.

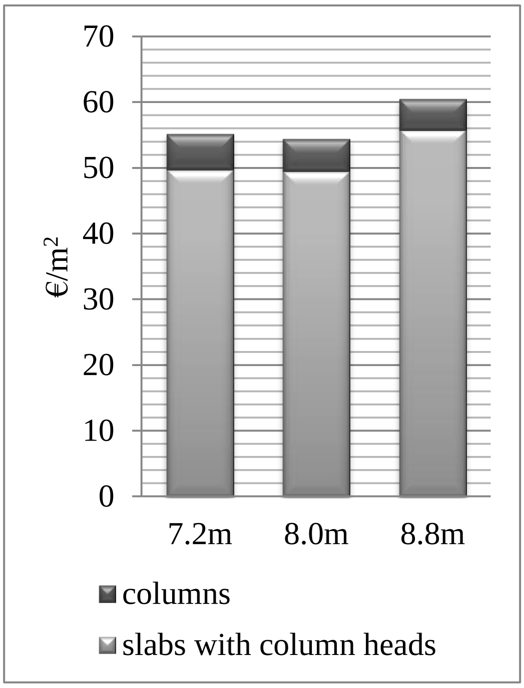


Figure 33: Total cost of slabs with column heads and columns per m2 of the total area of the building.

5.2.2. Cost structure of waffle slabs

The waffle slab is more economical for bigger spans, although the price per m^2 for span of 8.8m is lesser than for span of 8.0m. This can be result of using the simplified method for analysis. The reinforcement in the ribs of waffle slab of span 8.8m are better chosen and there is not a significant difference between resistance of every rib and applied bending moment. Within the simplified analysis that may play a serious role because the values of applied forces and moments stays the same in repeated panels, strips or beams. Therefore it is very important to what extend the resistance of a member is used. The cost of concrete per m2 of total area decreases with the growing span distance. The cost of formwork and labour oscillates between 16.58*€/m^2* and 19.77*€/m^2*. The cost structure divided into different resources of those cost is presented in the Figure 34.

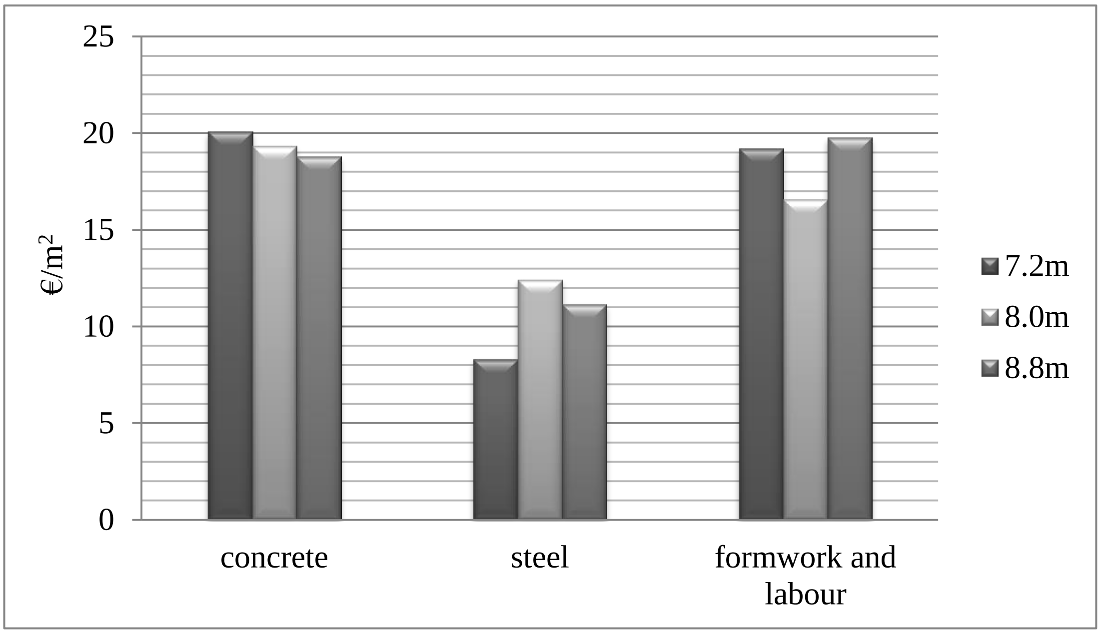


Figure 34: Comparison of costs of materials and formwork and labour per m^2 of total area of building for waffle slabs.

The cost of columns reaches from 11.03% of the cost of slabs for the span of 8.8m to 12.89% for the span of 7.2m. In the Figure 35 total cost of waffle slabs and columns per m^2 of the total area of the building is shown. The total cost of building slabs and columns are estimated to equal the value of 46.07*€/m^2* for the span of 7.2m and 49.57*€/m^2* for the span of 8.0m.

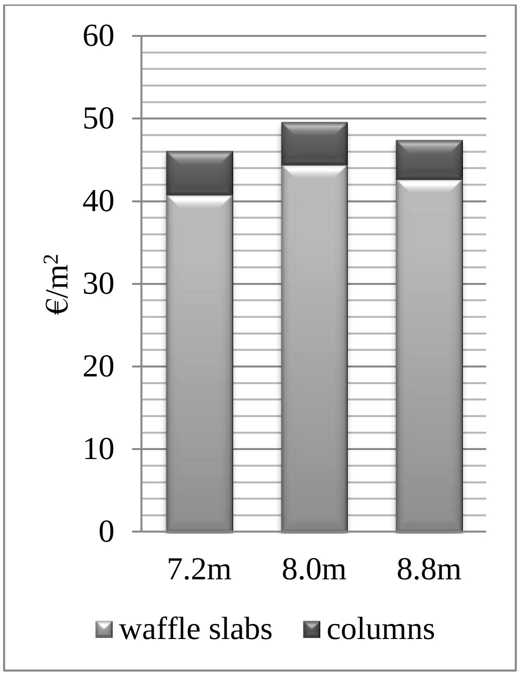


Figure 35: Total cost of waffle slabs and columns per m^2 of the total area of the building.

5.2.3. Cost structure of flat plates

Flat plates were designed with the grid of columns of spans from 5.6m to 7.2m. The cost of concrete per m^2 raises with the increasing spans from the price of 21.62*€/m^2* to 26.59*€/m^2*. It is connected with the height of the slab which must be increased with approximately 3cm for every additional 0.8m of span within the analyzed range. The cost of steel is very low for the span of 5.6m. It is only 7.40*€/m^2* . For columns grid of bigger spans the cost of steel grows significantly but for both spans of 6.4m and 7.2m stays almost at the same level of 14.43*€/m^2* to 14.64*€/m^2*. The lower labour cost for the span of 5.6m is connected with relatively low volume of steel and concrete required. The cost structure divided into different resources is presented in the Figure 36.

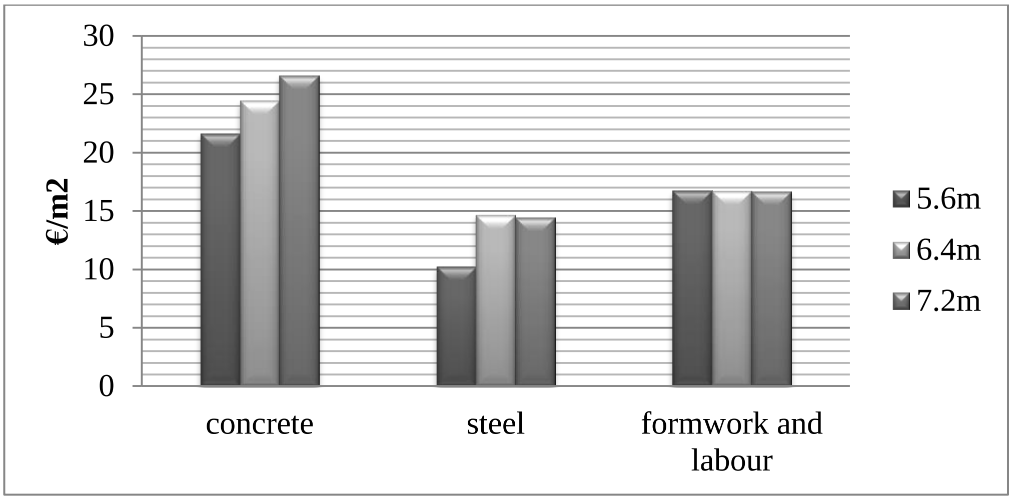


Figure 36: Comparison of costs of materials and formwork and labour per m^2 of total area of building for flat plates.

The cost of columns reaches from 11.6% of the cost of slabs for the span of 7.2m to 15.4% for the span of 5.6m. In the Figure 37 total cost of waffle slabs and columns per m^2 of the total area of the building is shown. The total cost of building slabs and columns are estimated to equal the value from 48.06*€/m²* for the span of 5.2m to 56.15*€/m²* for the span of 7.2m.

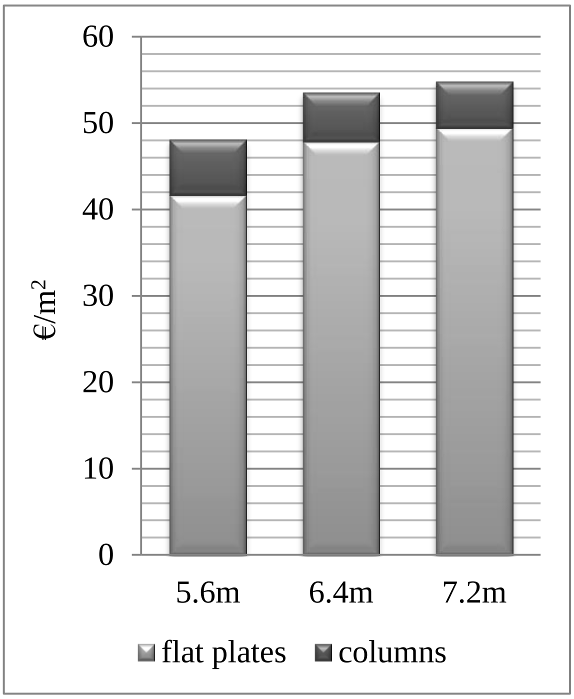


Figure 37: Total cost of flat plates and columns per m^2 of the total area of the building

5.2.4. Cost structure of slabs with beams

Slabs with beams were designed with the grid of columns of spans from 5.6m to 7.2m. Cost of building the slab for the span of 5.6m per m^2 of total area is very close to the price for span of 7.2 m. The cost of concrete grows with the increasing spans form the price of 20.22*€/m²* to 25.12*€/m²*. The cost of steel is rather high for the span of 5.6m. It is 26.08*€/m²* . The cost of columns decreases with the increasing span from 9.99*€/m²* for span of 5.6m to 6.81*€/m²* for the span of 7.2m. The labour cost for sll of the spans are similar and fall within the ambit of 16.18*€/m²* to 17.47*€/m²*. The cost structure divided into different resources is presented in the Figure 38.

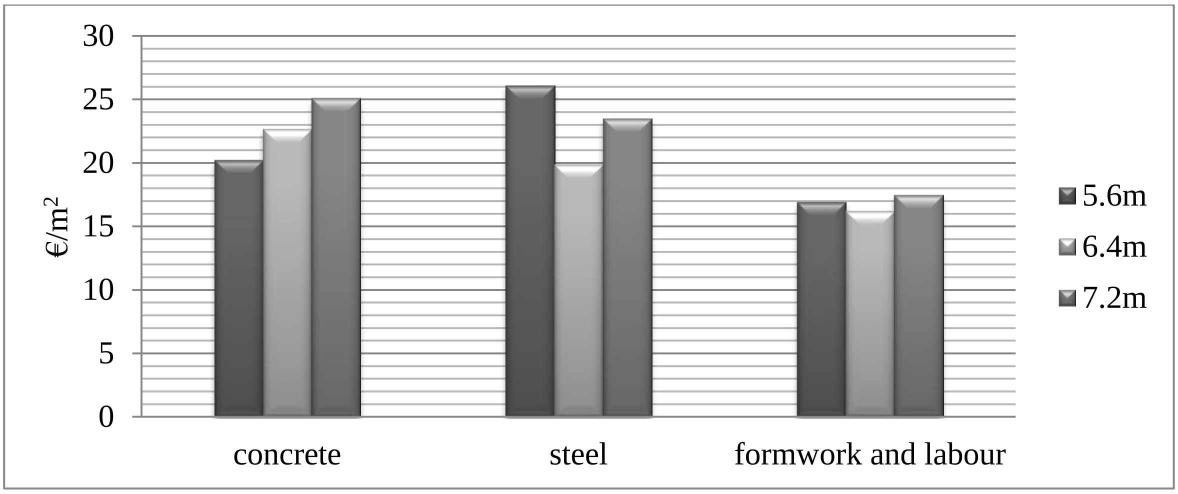


Figure 38: Comparison of costs of materials and formwork and labour per m^2 of total area of building for slabs with beams

The cost of columns supporting slabs with beams reaches from 12.0% of the cost of slabs for the span of 7.2m to 18.4% for the span of 5.6m. In the Figure 39 total cost of slabs with beams and columns per m^2 of the total area of the building is shown. The total cost of building slabs and columns are estimated to equal the value from 58.24*€/m²* for the span of 6.4m to 64.18*€/m²* for the span of 5.6 m.

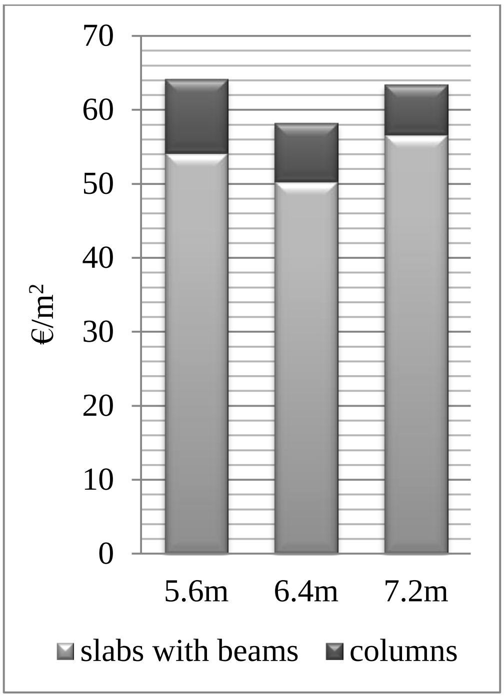


Figure 39: Total cost of slabs with beams and columns per m^2 of the total area of the building

5.2.5. Comparison of different slab systems

As shown in Figure 40 for span of 7.2m the price for building one m^2 of slab differs significantly among different systems of flat slabs. The most economical is waffle slab and the most expensive solution for this span is slab with beams which additionally have big dimensions. The internal beams have width of 40cm what makes it very difficult to hide them in internal walls. Flat plate and slab with column heads have similar cost in the considered example.

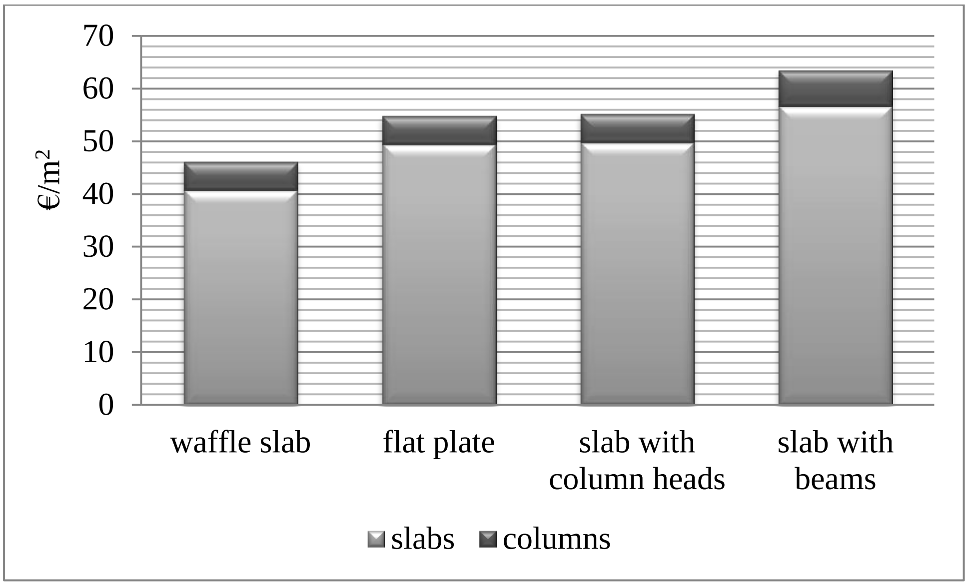


Figure 40: Total costs of slabs and columns per m^2 of total area of building for span of 7.2m.

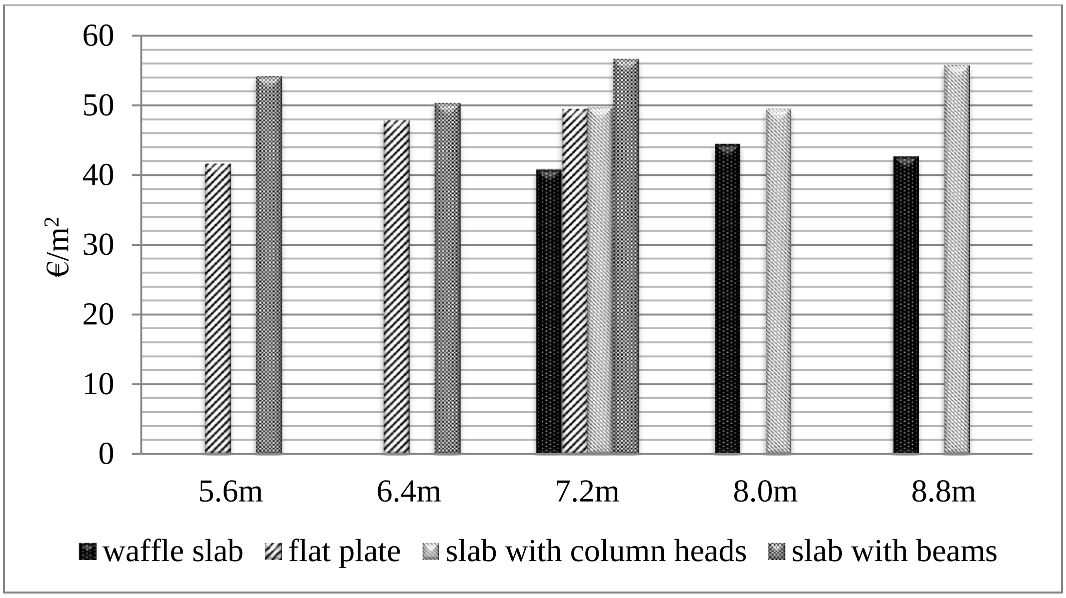


Figure 41: Comparison of costs of materials and formwork and labour per m^2 of total area of building for all considered slabs.

The highest price per m^2 of total area is reached by the slab with beams for the span of 7.2m and it is equal to 56.66*€/m^2* . This type of slab is very expensive in all of considered cases. It is the most economical for the span of 6.4m but still is 5.14% more expensive than flat plate. For the span of 5.6m the difference in cost is even bigger and reaches 30%.

The lowest price per m^2 of total area is reached by the waffle slab for the span of 7.2m and it is equal to 40.80*€/m²* . For bigger spans the price for waffle slab does not change significantly and reaches the value of 44.43€/m^2 for span of 8.0m what is 8.88% more than the lowest price . This type of slab is very economical in all of considered cases especially when compared to slab with column heads for the span of 8.8m. For this span the solution with column heads costs 30.7% more.

6. FINAL CONCLUSIONS

For the slabs a significant part of the total volume of concrete for a bulding is used as they have important roles in the structures. The choice of slab system must be performed taking into account several aspects as economic evaluation, functionality, implementation, and the interaction with other structural elements of the building. Therefore, the flat reinforced concrete slabs constitute one of the best alternatives for the construction of floors of buildings.

In the era of finite elements method usage of simplified methods may be helpful to evaluate the dimensions of structural members. Knowledge of the simplified methods helps to understand the mechanics of building and to find possible errors in computer supported design. The simplified methods are methods for quick and easy use, however, these are limited to slabs with repetitive supports and uniform loads.

The conclusion from the conducted calculation may be drawn that the cost of concrete as a construction material is higher for the flat plate than for the slab with beams for every span considered respectively but the cost in steel between these two systems is significantly different. The slab with beams requires much more steel than flat plate what makes flat plate more economical solution.

The comparison of costs of different types of flat slabs and columns made in this dissertation was not meant to be very complex. The simplified methods used for designing the structural members cannot be used for complex analysis but for simple grid of columns were enough precise to shed light on different aspects which has to be take into consideration and make the difference in the total cost. Therefore, besides knowledge acquired in process of writing the dissertation, the objectives of it were achieved.

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APPENDIX A.1

WAFFLE SLAB

7.2m x7.2m

Materials

Concrete	f_{ck}	f_{cm}	f_{ctm}	E_{cm}	ρ_c	γ_c	**Steel**	f_{yk}	E_s	γ_s
C30/37	MPa	MPa	MPa	GPa	kN/m^3	-	**A500**	MPa	GPa	-
	30	38	2.9	33	25.0	1.5		500	210	1.1 5

Actions

Category of loaded area: A - Areas for domestic and residential activities

Permanent actions	g_k kN/m^2
Imposed load	5.0
External walls	3.8

Variable actions	q_k kN/m^2
Imposed load	2.0

Pre-dimensioning of slab

concrete lightly stressed: ρ = 0,5%	***l***	***l/d***	***d***
	m	-	cm
end span of two-way spanning slab continouos over one long side	7.2	26	27.7
interior span of two-way spanning slab	7.2	30	24.0

l - span

d - effective depth of a cross-section

Dimensioning of slab

Height of slab

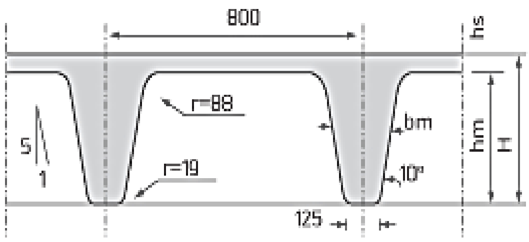
l_x	l_y	l/d	$Ø_{stirrup}$	$Ø_{bottom}$	$Ø_{top}$	d	c_{nom}	h
m	m	-	mm	mm	mm	mm	mm	mm
7.2	7.2	24.91	6	20	16	**289**	20	325

Area of slab

B_x	B_y	A
m	m	m^2
28.8	14.4	414.72

Geometry of recuperative molds

Mold		325-75	
Distance between ribs		800	mm
Height of mold	hm	250	mm
Thickness of topping slab	hs	75	mm
Total height	H	325	mm
Mean width of rib	bm	177	mm
Area of section		1010	cm^2
Inertia		79426	cm^4
Dead load		5.05	kN/m^2
Volume of concrete		0.177	m^3/m^2



The waffle slab has sufficient torsional stiffness, according to Eurocode 2, if:

- the rib spacing does not exceed 1500 mm
- the depth of the rib below the flange does not exceed 4 times its width.
- the depth of the flange is at least 1/10 of the clear distance between ribs or 50 mm, whichever is the greater.
- transverse ribs are provided at a clear spacing not exceeding 10 times the overall depth of the slab

Geometry of column heads

slab over:	$0.15\ L_x$	$0.15\ L_y$	No of molds		$L_{head,x}$	$L_{head,y}$	$L_{rib,x}$	$L_{rib,y}$	Edge beams	
			x	y					b_x	b_y
	m	m	-	-	m	m	m	m	m	m
ground floor 1st floor	1.08	1.08	4	4	3.20	3.20	4.00	4.00	0.25	0.25
2nd, 3rd	1.08	1.08	4	4	3.20	3.20	4.00	4.00	0.25	0.25
4th, 5th, 6th	1.08	1.08	4	4	3.20	3.20	4.00	4.00	0.25	0.25

Preidimensioning of columns

column on:	position of column	A_{inf}	β	c
		m^2	-	m
1st floor	internal	51.84	1.15	0.55
	edge	25.92	1.40	0.45
	corner	12.96	1.50	0.35
2nd, 3rd	internal	51.84	1.15	0.50
	edge	25.92	1.40	0.40
	corner	12.96	1.50	0.30
4th, 5th, 6th	internal	51.84	1.15	0.40
	edge	25.92	1.40	0.30
	corner	12.96	1.50	0.30

β - recommended values where adjacent spans do not differ more than 25%

Ultimate limit state

Bending

Bending moments

Direction x

strip	$M_{Ed,x,A}$	$M_{Ed,x,1}$	$M_{Ed,x,B}$	$M_{Ed,x,2}$	$M_{Ed,x,C}$
	kNm	kNm	kNm	kNm	kNm
column	189.29	278.14	454.29	259.59	412.99
middle	81.12	185.42	194.70	173.06	177.00

Direction y

strip	$M_{Ed,A}$	$M_{Ed,1}$	$M_{Ed,B}$
	kNm	kNm	kNm
column	189.29	278.14	504.77
middle	81.12	185.42	216.33

Calculation of bottom reinforcement

Minimum reinforcement in ribs of waffle slab

b	concrete cover	d	$A_{s,min}$	$A_{s,min,bottom}$	
mm	mm	mm	cm²/rib	-	cm²
125	20	289	0.54	2Ø8	1.01

$$A_{s,\min} = 0,26\frac{f_{ctm}}{f_{yk}}b_t d \geq 0,0013 b_t d$$

Distance between bars

$$s \geq máximo\{\phi ; d_g + 5mm ; 20mm\}$$

s ≥ 0.02m

Reinforcement in direction x

$$\mu = \frac{M_{Ed}}{bd^2 f_{cd}}$$

$$\omega = 0,973(1-\sqrt{1-2,056\mu})$$

$$A_s = \frac{\omega b d f_{cd}}{f_{yd}}$$

		M_{Ed}	μ	ω	$A_{s,min}$	$A_{s,min}$/rib	A_s/rib	$A_{s,bottom}$
		kNm	-	-	cm²/m	cm²	cm²	-
$M_{Sd,1}$	column strip	278.14	0.046	0.047	6.30	5.04	6.28	2Ø20
	middle strip	185.42	0.031	0.031	4.17	3.33	4.02	2Ø16
$M_{Sd,2}$	column strip	259.59	0.043	0.044	5.87	4.70	6.28	2Ø20
	middle strip	173.06	0.029	0.029	3.89	3.11	4.02	2Ø16

Reinforcement in direction y

		M_{Ed}	μ	ω	$A_{s,min}$	$A_{s,min}$/rib	A_s/rib	$A_{s,bottom}$
		kNm/m	-	-	cm²/m	cm²	cm²	-
$M_{Sd,1}$	column strip	278.14	0.046	0.047	6.30	5.04	6.28	2Ø20
	middle strip	185.42	0.031	0.031	4.17	3.33	4.02	2Ø16

Calculation of top reinforcement

Minimal reinforcement in topping slab

$$A_{s,min} = 0{,}26\frac{f_{ctm}}{f_{yk}}b_t d \geq 0{,}0013 b_t d$$

b	*concrete cover*	*d*	*$A_{s,min}$*	*$0.0013b_td$*	*$A_{s,min,top}$*	
mm	mm	mm	cm²/m	cm²/m	-	cm²
1000	20	291	4.39	3.78	Ø8//0.10	5.03

Top reinforcement in direction x

		M_{Ed}	μ	ω	A_s	$A_{s,top}$	
		kNm/m	-	-	cm²/m	-	cm²/m
$M_{Sd,A}$	column strip	189.29	0.031	0.032	4.23	Ø8//0.10	5.03
	middle strip	81.12	0.013	0.013	1.79	Ø8//0.10	5.03
$M_{Sd,B}$	column strip	454.29	0.075	0.078	10.39	Ø12//0.10	11.31
	middle strip	194.70	0.032	0.032	4.35	Ø8//0.10	5.03
$M_{Sd,C}$	column strip	412.99	0.068	0.070	9.41	Ø12//0.10	11.31
	middle strip	177.00	0.029	0.029	3.95	Ø8//0.10	5.03

Top reinforcement in direction y

		M_{Ed}	μ	ω	A_s	$A_{s,top}$	
		kNm/m	-	-	cm²/m	-	cm²/m
$M_{Sd,A}$	column strip	189.29	0.031	0.032	4.23	Ø8//0.10	5.03
	middle strip	81.12	0.013	0.013	1.79	Ø8//0.10	5.03
$M_{Sd,B}$	column strip	504.77	0.083	0.087	11.60	Ø16//0.15	13.40
	middle strip	216.33	0.035	0.036	4.84	Ø8//0.10	5.03

Shear force

Verification of shear resistance shoudl be checked in proximity of drop panels.

Shear force in ribs according to Tesoro (1991)

n	***K***	***B = L***	P_1	Q_a	V_{Ed}
-	-	m	kN/m²	kN	kN
8	0.9	7.2	20.55	213.06	53.27
8	1.1	7.2	20.55	260.41	65.10
8	1.0	7.2	20.55	236.74	59.18
8	1.0	7.2	20.55	236.74	59.18

In direction x

support	A	B_{left}	B_{right}	C
	kN	kN	kN	kN
V_{Ed}	53.27	65.10	59.18	59.18

In direction y

support	A	B
	kN	kN
V_{Ed}	53.27	65.10

Members not requiring design shear reinforcement

$$V_{Rd,c} = \left[C_{Rd,c}\, k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp}\right] b_w d \geq (v_{min} + k_1 \sigma_{cp}) b_w d$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2,0 \qquad \rho_l = \frac{A_{sl}}{bd} \leq 0,02 \qquad v_{min} = 0.035 k^{3/2} f_{ck}^{1/2}$$

f_{ck}	d	$C_{Rd,c}$	k	b_w	$A_{s,bottom}$	A_{sl}	ρ_l	v_{min}	$V_{Rd,c}$	$v_{min} b_w d$
MPa	mm	-	-	m	-	cm²	-	kPa	kN	kN
30	289	0.12	1.83	0.135	2Ø16	4.02	0.0103069	475.31	26.92	18.54
30	289	0.12	1.83	0.135	2Ø20	6.28	0.0161045	475.31	31.24	18.54

Calculation of shear reinforcement

Maximum longitudinal spacing of the stirrups assemblies

d	α	cotα	$s_{l,max}$
m	°	-	m
0.289	90	0	0.21675

$$s_{l,max} \leq 0{,}75d(1$$

The transverse spacing of the legs in a series of shear links

$$s_{t,max} = 1{,}5d$$

d	$s_{t,max}$
m	m
0.289	0.4335

Minimal reinforcement in a rib

$$\rho_{w,\min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yk}} \qquad \frac{A_{sw}}{s}$$

f_{ck}	f_{yk}	$\rho_{w,min}$	α	b_w	$A_{sw,min}/s$	A_{sw}/s	
MPa	MPa	-	°	m	cm²/m	cm²/m	
30	500	0.00088	90	0.135	1.18	2.83	Ø6 // 0.20

Shear reinforcement in ribs

$$\upsilon_1 = 0.6\left[1 - \frac{f_{ck}}{250}\right] \qquad V_{Rd,s} = \frac{A_{sw}}{s} z\, f_{ywd} \cot\theta \qquad V_{Rd,\max} = \frac{\alpha_{cw} b_w z\, v_1 f_{cd}}{\cot\theta + \tan\theta}$$

α_{cw}	b_w	d	z	v_1	f_{cd}	$\cot\Theta$	$V_{Rd,max}$
-	m	m	m	-	MPa	-	kN
1	0.135	0.292	0.263	0.6	20	2.5	146.81

V_{Ed}	f_{ywd}	A_{sw}/s	assumed (2 legs)	
				A_{sw}/s
kN	MPa	cm²/m	-	cm²/m
53.27	435	1.86	Ø6 // 0.20	2.83
65.10	435	2.28	Ø6 // 0.20	2.83
59.18	435	2.07	Ø6 // 0.20	2.83

Minimum shear reinforcement should be used in every section where bigger is not required.

Punching

Vertical force

column	V_{Ed}
	kN
internal	901.37
edge	543.02
corner	317.68

Punching shear resistance without shear reinforcement $v_{Rd,c}$

$$v_{Rd,c} = C_{Rd,c}\, k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp} \geq v_{min} + k_1 \sigma_{cp}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2{,}0$$

$$\rho_l = \sqrt{\rho_{lx}\, \rho_{ly}} \leq 0{,}02$$

$$\rho_{lx} = \frac{A_{sl,x}}{b_y d} \leq 0{,}02$$

$$\rho_{ly} = \frac{A_{sl,y}}{b_x d} \leq 0{,}02$$

$$d_{eff} = \frac{d_x + d_y}{2}$$

$$v_{min} = 0.035 k^{3/2} f_{ck}^{1/2}$$

Punching shear resistance without shear reinforcement

frame in direction x	support	position of column	$A_{s,top,x}$	$A_{s,top,y}$	$\rho_{l,head}$	$v_{Rd,c}$	v_{min}
			cm²/m	cm²/m	-	kPa	kPa
exterior	A	corner	5.03	5.03	1.73E-06	38.04	475.31
	B	edge	11.31	5.03	2.59E-06	43.55	
	C	edge	11.31	5.03	2.59E-06	43.55	
interior	A	edge	5.03	13.40	2.82E-06	44.80	
	B	internal	11.31	13.40	4.23E-06	51.28	
	C	internal	11.31	13.40	4.23E-06	51.28	

Maximum shear stress

Corner columns

$$v_{Ed} = \beta \frac{V_{Ed}}{u_i d}$$

slab above:	β	V_{Ed}	u_i	v_{Ed}
	-	kN	m	kPa
ground, 1st	1.50	317.68	0.968	1703.6
2nd ÷ 6th	1.50	317.68	0.868	1899.9

Edge columns

slab above:	β	V_{Ed}	u_1	v_{Ed}
	-	kN	m	kPa
ground, 1st	1.40	543.02	1.736	1499.97
2nd ÷ 3rd	1.40	543.02	1.586	1641.86
4th ÷ 6th	1.40	543.02	1.436	1813.39

Internal columns

support	slab above:	β	V_{Ed}	u_1	v_{Ed}
		-	kN	m	kPa
internal B	ground, 1st	1.15	901.37	2.171	1634.80
	2nd, 3rd	1.15	901.37	1.971	1800.65
	4th, 5th	1.15	901.37	1.771	2003.95
internal C	ground, 1st	1.15	901.37	2.171	1634.80
	2nd, 3rd	1.15	901.37	1.971	1800.65
	4th, 5th	1.15	901.37	1.771	2003.95

Calculation of punching reinforcement

There is an assumption that:
$v_{Ed} = v_{Rd,cs}$

Corner columns

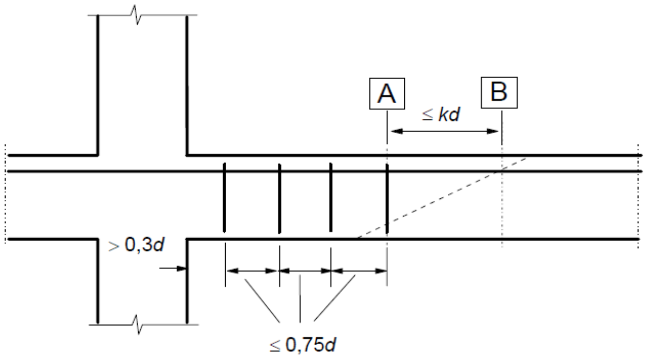
slab above:	u_1	$v_{Rd,cs}$	$v_{Rd,c}$	d	$f_{ywd,ef}$	α	A_{sw} / s_r
	m	kPa	kPa	m	MPa	°	cm²/m
ground, 1st	0.968	1703.6	475.31	0.291	322.75	90	26.93
2nd ÷ 5th	0.868	1899.9	475.31	0.291	322.75	90	27.67

Edge columns

slab above:	u_1	$v_{Rd,cs}$	$v_{Rd,c}$	d	$f_{ywd,ef}$	α	A_{sw} / s_r
	m	kPa	kPa	m	MPa	°	cm²/m
ground 1st	1.736	1500.0	475.31	0.291	322.75	90	41.00
2nd ÷ 3rd	1.586	1641.9	475.31	0.291	322.75	90	42.10
4th ÷ 5th	1.436	1813.4	475.31	0.291	322.75	90	43.21

Internal columns

support	slab above:	β	V_{Ed}	u_1	$v_{Rd,cs}$	$v_{Rd,c}$	d	$f_{ywd,ef}$	α	A_{sw} / s_r
		-	kN	m	kPa	kPa	m	MPa	°	cm²/m
internal 1	ground 1st	1.15	901.37	2.171	1634.80	475.31	0.291	322.75	90	57.34
	2nd, 3rd	1.15	901.37	1.971	1800.65	475.31	0.291	322.75	90	58.81
	4th, 5th,	1.15	901.37	1.771	2003.95	475.31	0.291	322.75	90	60.28
internal 2	ground 1st	1.15	901.37	2.171	1634.80	475.31	0.291	322.75	90	57.34
	2nd, 3rd	1.15	901.37	1.971	1800.65	475.31	0.291	322.75	90	58.81
	4th, 5th	1.15	901.37	1.771	2003.95	475.31	0.291	322.75	90	60.28



		assumed distance
	m	m
d	0.291	
0.3d	0.087	0.1
0.75d	0.218	0.2
kd	0.437	

$$u_{out} = \frac{\beta V_{Ed}}{v_{Rd,c} d}$$

The recommended value for *k* is 1.5.

Punching shear reinforcement should be placed between the column and *kd* inside the control perimeter at which shear reinforcement is no longer required.

Calculation of punching reinforcement

storey	$u_{out,ef}$	r	c	s_r	A_{sw}	A_{sw} / s_r	min A_{sw} / s_r
	m	m	m	m	cm²	cm²/m	cm²/m
Corner columns							
1st, 2nd	3.45	1.97	0.35	0.20	5.65	28.27	26.93
2nd ÷ 6th	3.45	2.00	0.30	0.20	5.65	28.27	27.67
Edge columns							
1st, 2nd	5.50	1.62	0.40	0.20	9.05	45.24	41.00
3rd, 4th	5.50	1.64	0.35	0.20	9.05	45.24	42.10
5th, 6th	5.50	1.65	0.30	0.20	9.05	45.24	43.21
Internal columns							
1st, 2nd	7.49	1.19	0.55	0.20	13.57	67.86	57.34
3rd, 4th	7.49	1.19	0.45	0.20	13.57	67.86	58.81
5th, 6th	7.49	1.19	0.35	0.20	13.57	67.86	60.28

storey	reinforcement per perimeter	Number of perimeters	Verification of the distance* ≤ kd=0.437m	Verification of the spacing[1] ≤ 2d=0.582m	$r_{assumed}$	$r_{obligatory}$
Corner columns						
1st, 2nd	5#12	8	0.14	0.52	1.68	1.53
2nd ÷ 6th	5#12	8	0.08	0.52	1.65	1.57
Edge columns						
1st, 2nd	8#12	6	0.11	0.51	1.30	1.19
3rd, 4th	8#12	6	0.07	0.50	1.28	1.20
5th, 6th	8#12	6	0.03	0.49	1.25	1.22
Internal columns						
1st, 2nd	12#12	3	0.02	0.40	0.78	0.76
3rd, 4th	12#12	4	0.17	0.48	0.93	0.76
5th, 6th	12#12	4	0.12	0.45	0.875	0.76

* - the distance between the outermost perimeter of shear reinforcement and the control perimeter u_{out}

[1] - the spacing of link legs around a perimeter should not exceed 2d

Edge beams

Minimum reinforcement

$$A_{s,\min} = 0,26\frac{f_{ctm}}{f_{yk}}b_t d \geq 0,0013 b_t d$$

b	concrete cover	*d*	$A_{s,min}$	$0.0013b_t d$		A_s
mm	mm	mm	cm²/m	cm²/m	-	cm²
250	20	366	1.38	1.19	2 Ø12	2.26

Bottom reinforcement

	M_{Ed}	μ	ω	$A_{s,min}$	A_s	
	kNm	-	-	cm^2	-	cm^2/m
V_1	28.75	0.069	0.071	2.38	2Ø16	4.02
V_2	23.00	0.055	0.057	1.89	2Ø12	2.26
V_3	28.75	0.069	0.071	2.38	2Ø16	4.02

$$\mu = \frac{M_{Ed}}{bd^2 f_{cd}}$$

$$\omega = 0{,}973(1 - \sqrt{1 - 2{,}056\mu})$$

$$A_s = \frac{\omega b d f_{cd}}{f_{yd}}$$

Top reinforcement

		M_{Ed}	μ	ω	$A_{s,min}$	A_s	
		kNm	-	-	cm^2	-	cm^2/m
V_1	edge	27.60	0.066	0.069	2.28	2Ø16	4.02
	internal	48.30	0.116	0.124	4.11	2Ø20	6.28
V_2	internal	41.40	0.099	0.105	3.48	2Ø16	4.02
	internal	41.40	0.099	0.105	3.48	2Ø16	4.02
V_3	edge	27.60	0.066	0.069	2.28	2Ø16	4.02
	internal	48.30	0.116	0.124	4.11	2Ø20	6.28

Verification of shear force should be defined in proximity of column head.

	support	V_{Ed}
		kNm
V_1	edge	51.91
	internal	62.29
V_2	internal	57.10
	internal	57.10
V_3	edge	51.91
	internal	62.29

Members not requiring design shear reinforcement

$$V_{Rd,c} = \left[C_{Rd,c}\, k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp}\right] b_w d \geq (v_{\min} + k_1 \sigma_{cp}) b_w d$$

$$k = 1 + \sqrt{\frac{200}{d} \leq 2{,}0} \qquad \rho_l = \frac{A_{sl}}{bd} \leq 0{,}02 \qquad v_{\min} = 0.035 k^{3/2} f_{ck}^{\ 1/2}$$

Beam	f_{ck}	d	$C_{Rd,c}$	k	b	A_{sl}	ρ_l	v_{min}	$V_{Rd,c}$	$v_{min} b_w d$
	MPa	m	-	-	m	cm²	-	kPa	kN	kN
V_1, V_2, V_3	30	0.366	0.12	1.02	0.25	3.77	0.0041201	198.46	25.98	18.16

Calculation of shear reinforcement

Maximum longitudinal spacing of the stirrups assemblies

$$s_{l,max} \leq 0{,}75 d (1 + cot\alpha)$$

d	α	cotα	$s_{l,max}$
m	°	-	m
0.289	90	0	0.21675

Minimum reinforcement

$$\rho_{w,\min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yk}} \qquad \frac{A_{sw}}{s}$$

f_{ck}	f_{yk}	$\rho_{w,min}$	α	b_w	$A_{sw,min}/s$	A_{sw}/s		
MPa	MPa	-	°	m	cm²/m	cm²/m	-	
30	500	0.00088	90	0.25	2.19	2.83	Ø6 // 0.20	2 legs

Shear reinforcement

$$\upsilon_1 = 0.6\left[1 - \frac{f_{ck}}{250}\right] \qquad V_{Rd,s} = \frac{A_{sw}}{s} z\, f_{ywd} \cot\theta \qquad V_{Rd,\max} = \frac{\alpha_{cw} b_w z\, v_1 f_{cd}}{\cot\theta + \tan\theta}$$

f_{ck}	α_{cw}	b_w	d	z	v_1	f_{cd}	$\cot\Theta$	$V_{Rd,max}$
MPa	-	m	m	m	-	MPa	-	kN
30	1	0.25	0.289	0.260	0.528	20	2.5	236.78

Shear reinforcement

	support	V_{Ed}	A_{sw}/s	Assumed reinforcement		
		kN	cm²/m	cm²/m	-	
V_1	edge	51.91	1.84	2.83	Ø6 // 0.20	2 legs
	internal	62.29	2.20			
V_2	internal	57.10	2.02			
	internal	57.10	2.02			
V_3	edge	51.91	1.84			
	internal	62.29	2.20			

Assumed shear reinforcement: Ø6 // 0.20

APPENDIX A.2

SLAB WITH COLUMN HEADS
7.2m x7.2m

Materials

Concrete	f_{ck}	f_{cm}	f_{ctm}	E_{cm}	ρ_c	γ_c	**Steel**	f_{yk}	E_s	γ_s
C30/37	MPa	MPa	MPa	GPa	kN/m^3	-	**A500**	MPa	GPa	-
	30	38	2.9	33	25.0	1.5		500	210	1.15

Actions

Category of loaded area: A - Areas for domestic and residential activities

Permanent actions	g_k (kN/m^2)
Imposed load	5.0
External walls	3.8

Variable actions	q_k (kN/m^2)
Imposed load	2.0

	p_d (kN/m^2)
total design load	**17.33**

Pre-dimensioning of slab

concrete lightly stressed: ρ = 0,5%	***l***	***l/d***	***d***
	m	-	cm
Slab supported on columns without beams	7.2	24	30.0

l - span
d - effective depth of a cross-section

Dimensioning of slab

Area of slab

B_x	B_y	A
m	m	m^2
28.8	14.4	414.72

Geometry of column heads

$l_x/3$	$l_y/3$	$L_{head,x}$	$L_{head,y}$	$L_{slab,x}$	$L_{slab,y}$	Edge beams b_x	Edge beams b_y
m	m	m	m	m	m	m	m
2.40	2.40	2.40	2.40	4.8	4.8	0.25	0.25

Geometry of slab

l/d	d_{slab}	h_{slab}	h_{head}	h_H	$0.03l$	Verification
-	m	m	m	m	m	$h_H < 0.03l$
30	**0.16**	**0.20**	**0.40**	0.20	0.216	correct

Predimensioning of columns

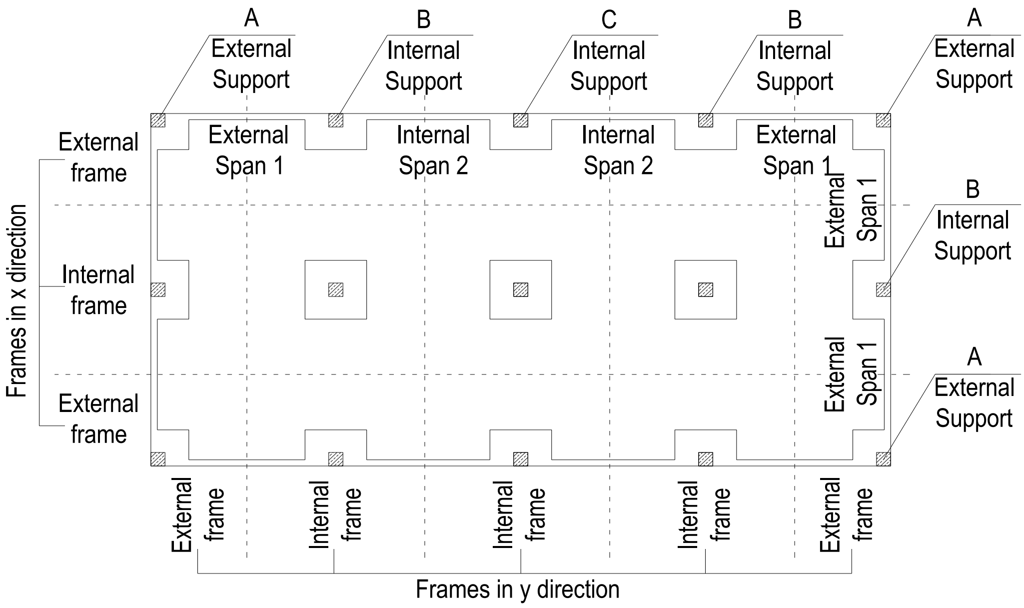
column on:	position of column	A_{inf}	β	c
		m^2	-	m
ground, 1st floor	internal	51.84	1.15	0.50
	edge	25.92	1.40	0.40
	corner	12.96	1.50	0.30
2nd, 3rd	internal	51.84	1.15	0.45
	edge	25.92	1.40	0.35
	corner	12.96	1.50	0.30
4th, 5th, 6th	internal	51.84	1.15	0.35
	edge	25.92	1.40	0.30
	corner	12.96	1.50	0.30

Reinforcement ratio assumed in columns: 2%

c - dimension of column side
β - recommended values where adjacent spans do not differ more than 25%

Ultimate limit state

Bending



Bending moments

Direction x

strip	$M_{Ed,x,A}$	$M_{Ed,x,1}$	$M_{Ed,x,B}$	$M_{Ed,x,2}$	$M_{Ed,x,C}$
	kNm	kNm	kNm	kNm	kNm
column	188.66	277.22	452.79	258.74	411.62
middle	80.85	184.81	194.05	172.49	176.41

Direction y

strip	$M_{Ed,A}$	$M_{Ed,1}$	$M_{Ed,B}$
	kNm	kNm	kNm
column	188.66	277.22	503.10
middle	80.85	184.81	215.61

Height of a slab area

$Ø_{bottom,x}$	$Ø_{bottom,y}$	$d_{slab,x}$	$d_{slab,y}$	c_{nom}	h
mm	mm	mm	mm	mm	mm
12	16	**158**	**172**	20	200

Calculation of bottom reinforcement

Minimum reinforcement

$$A_{s,\min} = 0{,}26\frac{f_{ctm}}{f_{yk}}b_t d \geq 0{,}0013 b_t d$$

b	concrete cover	***d***	***A**$_{s,min}$*	*0.0013b$_t$d*	***A**$_{s,min,bottom}$*	
mm	mm	mm	cm^2/m	cm^2/m	-	cm^2
1000	20	158	2.38	2.05	Ø8//0.20	2.51
1000	20	172	2.59	2.24	Ø8//0.20	2.51

Distance between bars

s ≥ 0.02m

$$s \geq máximo\{\phi ; d_g + 5mm ; 20mm\}$$

Reinforcement in direction x

$$\mu = \frac{M_{Ed}}{bd^2 f_{cd}} \qquad \omega = 0{,}973(1-\sqrt{1-2{,}056\mu}) \qquad A_s = \frac{\omega b d f_{cd}}{f_{yd}}$$

`

		M_{Ed}	μ	ω	$A_{s,min}$	$A_{s,bottom}$	
		kNm/m	-	-	cm^2/m	-	cm^2/m
$M_{Ed,1}$	column strip	277.22	0.154	0.169	12.28	Ø16//150	13.40
	middle strip	184.81	0.103	0.109	7.92	Ø12//125	9.05
$M_{Ed,2}$	column strip	258.74	0.144	0.157	11.38	Ø16//150	13.40
	middle strip	172.49	0.096	0.101	7.36	Ø12//150	7.54

Reinforcement in direction y

		M_{Ed}	μ	ω	$A_{s,min}$	$A_{s,bottom}$	
		kNm/m	-	-	cm^2/m	-	cm^2/m
$M_{Ed,1}$	column strip	277.22	0.130	0.140	11.10	Ø16//150	13.40
	middle strip	184.81	0.087	0.091	7.20	Ø12//150	7.54

Calculation of top reinforcement

Height of a column head area

$Ø_{top,x}$	$Ø_{top,y}$	$d_{head,x}$	$d_{head,y}$	c_{nom}	h_H
mm	mm	mm	mm	mm	mm
16	16	**356**	**372**	20	400

Minimum reinforcement

$$A_{s,min} = 0,26\frac{f_{ctm}}{f_{yk}}b_t d \geq 0,0013 b_t d$$

b	*concrete cover*	*d*	$A_{s,min}$	$0.0013b_td$	$A_{s,min,to}$	
mm	mm	mm	cm²/m	cm²/m	-	cm²
1000	20	372	5.61	4.84	Ø10//150	5.24
1000	20	365	5.50	4.75	Ø10//150	5.24

Top reinforcement in direction x

$$\mu = \frac{M_{Ed}}{bd^2 f_{cd}} \qquad A_s = \frac{\omega b d f_{cd}}{f_{yd}} \qquad \omega = 0,973(1-\sqrt{1-2,056\mu})$$

		M_{Ed}	b	μ	ω	A_s	$A_{s,top,x}$	
		kNm	m	-	-	cm²/m	-	cm²/m
$M_{Sd,A}$	column strip	188.66	3.6	0.021	0.021	3.42	Ø10//150	5.24
	middle strip	80.85		0.045	0.046	3.35	Ø10//150	5.24
$M_{Sd,B}$	column strip	452.79		0.050	0.051	8.35	Ø16//200	10.05
	middle strip	194.05		0.108	0.115	8.34	Ø16//200	10.05
$M_{Sd,C}$	column strip	411.62		0.045	0.046	7.57	Ø12//150	7.54
	middle strip	176.41		0.098	0.104	7.54	Ø12//150	7.54

Top reinforcement in direction y

		M_{Ed}	b	μ	ω	A_s	$A_{s,top,y}$	
		kNm	m	-	-	cm²/m	-	cm²/m
$M_{Sd,A}$	column strip	188.66	3.6	0.019	0.019	3.58	Ø10//150	5.24
	middle strip	80.85		0.038	0.039	3.65	Ø10//150	5.24
$M_{Sd,B}$	column strip	503.10		0.050	0.052	8.72	Ø16//200	10.05
	middle strip	215.61		0.101	0.107	9.08	Ø16//200	10.05

Punching

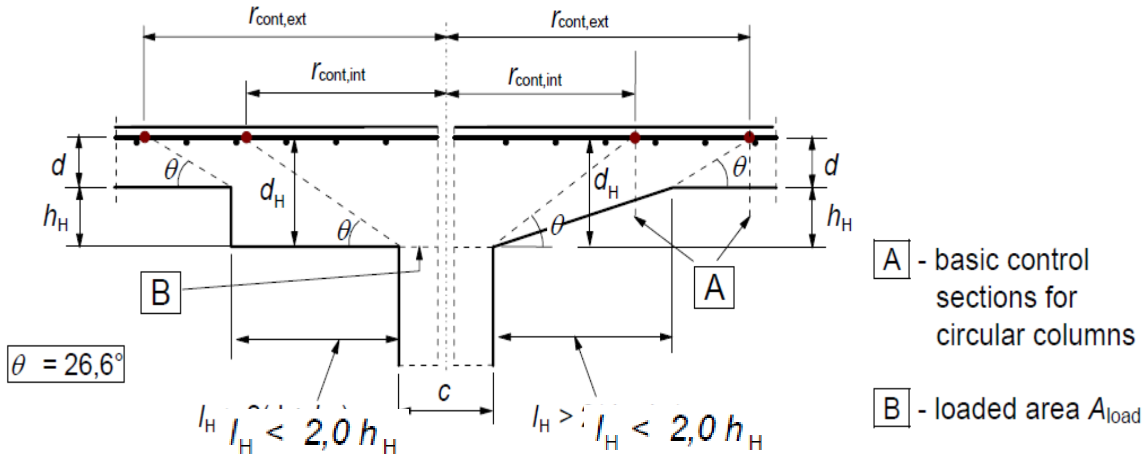
Vertical force

column position		internal	edge	corner
V_{Ed}	kN	894.24	539.46	315.90

Dimensions

h_{slab}	d	h_{head}	d_H	h_H	l_H	L_{head}
m	m	m	m	m	m	m
0.20	0.165	0.40	0.364	0.20	0.90÷1.05	2.40

For slabs with enlarged column heads where $l_H > 2h_H$ control sections both within the head and in the slab should be checked.



Punching shear resintance without shear reinforcement $v_{Rd,c}$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2{,}0$$

$$\rho_{lx} = \frac{A_{sl,x}}{b_y d} \leq 0{,}02$$

$$v_{Rd,c} = C_{Rd,c}\, k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp} \geq v_{min} + k_1 \sigma_{cp}$$

$$\rho_l = \sqrt{\rho_{lx}\, \rho_{ly}} \leq 0{,}02$$

$$\rho_{ly} = \frac{A_{sl,y}}{b_x d} \leq 0{,}02$$

$$d_{eff} = \frac{d_x + d_y}{2}$$

$$v_{min} = 0.035 k^{3/2} f_{ck}^{1/2}$$

k_{slab} calculated	k_{slab} assumed	d_{slab}	k_{head}	d_{head}	$C_{Rd,c}$
-	-		-		-
2.101	2.000	0.165	1.741	0.364	0.120

Punching shear resistance without shear reinforcement within the head

frame in direction x	support	position of column	$A_{s,top,x}$	$A_{s,top,y}$	head $\rho_{l,head}$	head $v_{Rd,c}$	head v_{min}	slab $\rho_{l,slab}$	slab $v_{Rd,c}$	slab v_{min}
			cm²/m	cm²/m	-	kPa	kPa	-	kPa	kPa
exterior	A	corner	5.24	5.24	0.001438	340.18	440.47	0.003173	508.65	542.22
	B	edge	10.05	5.24	0.001993	379.26		0.004397	567.07	
	C	edge	7.54	5.24	0.001726	361.50		0.003808	540.53	
interior	A	edge	5.24	10.05	0.001993	379.26		0.004397	567.07	
	B	internal	10.05	10.05	0.002762	422.81		0.006093	632.20	
	C	internal	7.54	10.05	0.002392	403.02		0.005277	602.61	

Verification of necessity of use of shear punching reinforcement in the column head

storey	c	l_H	u_1	β	V_{Ed}	v_{Ed}	$v_{Rd,c}$	Necessity of using shear punching reinforcement
	m	*m*	*m*	-	*kN*	*kPa*	*kPa*	
Corner columns								
1st, 2nd	0.30	1.05	1.74	1.50	315.90	746.63	440.47	necessary
3rd, 4th	0.30	1.05	1.74	1.50	315.90	746.63	440.47	
5th, 6th	0.30	1.05	1.74	1.50	315.90	746.63	440.47	
Edge columns								
1st, 2nd	0.40	1.00	3.49	1.40	539.46	595.01	440.47	necessary
3rd, 4th	0.35	1.03	3.34	1.40	539.46	621.76	440.47	
5th, 6th	0.30	1.05	3.19	1.40	539.46	651.02	440.47	
Internal columns B								
1st, 2nd	0.50	0.95	6.57	1.15	894.24	429.74	440.47	not necessary
3rd, 4th	0.45	0.98	6.37	1.15	894.24	443.23	440.47	necessary
5th, 6th	0.35	1.03	6.17	1.15	894.24	457.89	440.47	
Internal columns C								
1st, 2nd	0.50	0.95	6.57	1.15	894.24	429.74	440.47	not necessary
3rd, 4th	0.45	0.98	6.37	1.15	894.24	443.23	440.47	necessary
5th, 6th	0.35	1.03	6.17	1.15	894.24	457.89	440.47	

Verification of necessity of use of shear punching reinforcement in the slab

storey	c	l_H	u_2	β	V_{Ed}	v_{Ed}	$v_{Rd,c}$	Necessity of using shear punching reinforcement
	m	*m*	*m*	-	*kN*	*kPa*	*kPa*	
Corner columns								
1st, 2nd	0.30	1.05	3.22	1.50	315.90	892.32	542.22	necessary
3rd, 4th	0.30	1.05	3.22	1.50	315.90	892.32	542.22	
5th, 6th	0.30	1.05	3.22	1.50	315.90	892.32	542.22	
Edge columns								
1st, 2nd	0.40	1.00	5.84	1.40	539.46	784.21	542.22	necessary
3rd, 4th	0.35	1.03	5.84	1.40	539.46	784.21	542.22	
5th, 6th	0.30	1.05	5.84	1.40	539.46	784.21	542.22	
Internal columns B								
1st, 2nd	0.50	0.95	11.67	1.15	894.24	533.91	632.20	not necessary
3rd, 4th	0.45	0.98	11.67	1.15	894.24	533.91	632.20	
5th, 6th	0.35	1.03	11.67	1.15	894.24	533.91	632.20	
Internal columns C								
1st, 2nd	0.50	0.95	11.67	1.15	894.24	533.91	602.61	not necessary
3rd, 4th	0.45	0.98	11.67	1.15	894.24	533.91	602.61	
5th, 6th	0.35	1.03	11.67	1.15	894.24	533.91	602.61	

$v_{Rd,c}$ - punching shear resintance without shear reinforcemen

v_{Ed} - punching shear stress

$$v_{Ed} = \frac{\beta V_{Ed}}{u_2 d_{eff,slab}}$$

u_2 - length of control perimeter in the slab

Calculation of punching reinforcement

$$f_{ywd,ef} = 250 + 0.25d \leq f_{ywd}$$

$$\frac{A_{sw}}{s_r} = \frac{v_{Rd,cs} - 0.75 v_{Rd,c}}{1.5\, d\, f_{ywd,ef} \frac{1}{u_1 d} sen\,\alpha}$$

There is an assumption that: $v_{Ed} = v_{Rd,cs}$

Shear punching reinforcement in the column head

Corner columns

storey	u_1	$v_{Rd,cs}$	$v_{Rd,c}$	d	$f_{ywd,ef}$	α	A_{sw} / s_r
	m	kPa	kPa	m	MPa	°	cm²/m
1st, 2nd	1.744	746.6	440.47	0.364	341	90	14.19
3rd, 4th	1.744	746.6	440.47	0.364	341		14.19
5th, 6th	1.744	746.6	440.47	0.364	341		14.19

Edge columns

storey	u_1	$v_{Rd,cs}$	$v_{Rd,c}$	d	$f_{ywd,ef}$	α	A_{sw} / s_r
	m	kPa	kPa	m	MPa	°	cm²/m
1st, 2nd	3.487	595.0	440.47	0.364	341	90	18.04
3rd, 4th	3.337	621.8	440.47		341		19.01
5th, 6th	3.187	651.0	440.47		341		19.98

Internal columns

storey	u_1	$v_{Rd,cs}$	$v_{Rd,c}$	d	$f_{ywd,ef}$	α	A_{sw} / s_r
	m	kPa	kPa	m	MPa	°	cm²/m
Internal columns B							
3rd, 4th	6.374	443.2	440.47	0.364	341	90	14.07
5th, 6th	6.170	457.9	440.47		341		15.38
Internal columns C							
3rd, 4th	6.574	443.2	440.47	0.364	341	90	14.51
5th, 6th	6.374	457.9	440.47		341		15.89

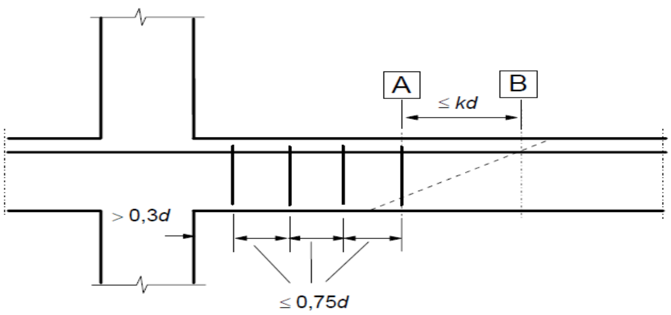
Shear punching reinforcement in the slab

Corner columns

storey	u_2	$v_{Rd,cs}$	$v_{Rd,c}$	d	$f_{ywd,ef}$	α	A_{sw} / s_r
	m	kPa	kPa	m	MPa	°	cm^2/m
1st, 2nd	3.218	892.3	542.22	0.165	291.25	90	35.78
3rd, 4th	3.218	892.3	542.22	0.165	291.25	90	35.78
5th, 6th	3.218	892.3	542.22	0.165	291.25	90	35.78

Edge columns

storey	u_2	$v_{Rd,cs}$	$v_{Rd,c}$	d	$f_{ywd,ef}$	α	A_{sw} / s_r
	m	kPa	kPa	m	MPa	°	cm^2/m
1st, 2nd	5.837	784.2	542.22	0.165	291.25	90	50.44
3rd, 4th	5.837	784.2	542.22	0.165	291.25	90	50.44
5th, 6th	5.837	784.2	542.22	0.165	291.25	90	50.44



	head	slab
	m	m
d	0.364	0.165
0.3d	0.109	0.050
0.5d	0.182	0.0825
0.75d	0.273	0.124
kd	0.546	0.248

The distance between the face of a support and the nearest shear reinforcement

The recommended value for *k* is 1.5.
Punching shear reinforcement should be placed between the column and *kd* inside the control perimeter at which shear reinforcement is no longer required.
The distance between the face of a support and the nearest shear reinforcement taken into account in the design should not exceed *d/2* (Eurocode 2, 2010)
For slabs with enlarged column heads where $l_H > 2h_H$ control sections both within the head and in the slab should be checked.

The control perimeter at which shear reinforcement is not required **u_{out}**

$d_{eff,head} = d_H$	$d_{eff,slab}$	$L_{head,x}$	$L_{head,y}$
m	m	m	m
0.364	0.165	2.40	2.40

Calculation of punching reinforcement in column heads

storey	u_{out} $u_{out,ef}$	r_{out}	s_r	A_{sw}	studs	Number of perimeters	A_{sw}/s_r	$(A_{sw}/s_r)_{MIN}$	Verification of the distance* ≤ kd=0.552m	Verification of the spacing[1] ≤ 2d=0.736m
	m	m	m	cm^2			cm^2/m	cm^2/m	m	m
Corner columns										
1st, 2nd	2.96	1.69	0.270	3.39	3#12	4	12.57	14.19	-0.002	0.591
3rd, 4th	2.96	1.69	0.275	3.39	3#12	4	12.34	14.19	0.013	0.599
5th, 6th	2.96	1.69	0.275	3.39	3#12	4	12.34	14.19	0.013	0.599
Edge columns										
1st, 2nd	4.71	1.37	0.27	3.93	5#10	2	14.54	18.04	-0.174	0.403
3rd, 4th	4.71	1.39	0.27	3.93	5#10	3	14.54	19.01	0.055	0.554
5th, 6th	4.71	1.40	0.27	5.65	5#12	3	20.94	19.98	0.014	0.539
Internal columns B										
3rd, 4th	6.41	1.90	0.27	5.65	5#12	5	20.94	14.07	0.060	0.628
5th, 6th	6.41	1.93	0.27	5.65	5#12	5	20.94	15.38	0.028	0.551
Internal columns C										
3rd, 4th	6.41	1.90	0.27	5.65	5#12	5	20.94	14.51	0.060	0.442
5th, 6th	6.41	1.93	0.27	5.65	5#12	5	20.94	15.89	0.028	0.402

* - the distance between the outermost perimeter of shear reinforcement and the control perimeter u_{out}

$$u_{out} = \frac{\beta V_{Ed}}{v_{Rd,c} d}$$

[1] - the spacing of link legs around a perimeter should not exceed 2d

Calculation of punching reinforcement within slab

storey	u_{out} $u_{out,ef}$	r_{out}	s_r	A_{sw}	studs	Number of perimeters	A_{sw}/s_r	$(A_{sw}/s_r)_{MIN}$	Verification of the distance* ≤ kd=0.248m	Verification of the spacing** ≤ 2d=0.33m
	m	m	m	cm²			cm²/m	cm²/m	m	m
Corner columns										
1st, 2nd	5.30	3.18	0.250	12.57	16#10	3	50.27	35.78	0.049	0.293
3rd, 4th	5.30	3.18	0.250	12.57	16#10	3	50.27	35.78	0.049	0.293
5th, 6th	5.30	3.18	0.250	12.57	16#10	3	50.27	35.78	0.049	0.293
Edge columns										
1st, 2nd	8.44	2.56	0.25	16.08	24#10	2	64.34	50.44	0.420	0.357
3rd, 4th	8.44	2.58	0.25	16.08	24#10	2	64.34	50.44	0.404	0.357
5th, 6th	8.44	2.59	0.25	16.08	24#10	2	64.34	50.44	0.388	0.357

$$u_{out} = \frac{\beta V_{Ed}}{v_{Rd,c} d}$$

* - the distance between the outermost perimeter of shear reinforcement and the control perimeter u_{out}

** - the spacing of link legs around a perimeter should not exceed 2d

Edge beams

Minimal reinforcement

b	concrete cover	***d***	$A_{s,min}$	*0.0013b* $0.0013 b_t d$		A_s
mm	mm	mm	cm²/m	cm²/m	-	cm²
250	20	368	1.39	1.20	2 Ø12	2.26

$$A_{s,min} = 0{,}26 \frac{f_{ctm}}{f_{yk}} b_t d \geq 0{,}0013 b_t d$$

Bottom reinforcement

$$\mu = \frac{M_{Ed}}{bd^2 f_{cd}}$$

$$\omega = 0{,}973(1 - \sqrt{1 - 2{,}056\mu})$$

$$A_s = \frac{\omega b d f_{cd}}{f_{yd}}$$

	M_{Ed}	μ	ω	$A_{s,min}$	A_s	
	kNm	-	-	cm²	-	cm²/m
V_1	28.68	0.043	0.044	1.40	2Ø12	2.26
V_2	22.94	0.035	0.035	1.12	2Ø12	2.26
V_3	28.68	0.043	0.044	1.40	2Ø12	2.26

Top reinforcement

		M_{Ed}	μ	ω	$A_{s,min}$	A_s	
		kNm	-	-	cm²	-	cm²/m
V_1	edge	27.53	0.042	0.042	1.35	2Ø12	2.26
	internal	48.18	0.073	0.076	2.40	2Ø16	4.02
V_2	internal	41.29	0.062	0.064	2.04	2Ø12	2.26
	internal	41.29	0.062	0.064	2.04	2Ø12	2.26
V_3	edge	27.53	0.042	0.042	1.35	2Ø12	2.26
	internal	48.18	0.073	0.076	2.40	2Ø16	4.02

Verification of shear force should be defined in proximity of column head.

	support	V_{Ed}
		kNm
V_1	edge	51.76
	internal	62.11
V_2	internal	56.94
	internal	56.94
V_3	edge	51.76
	internal	62.11

Members not requiring design shear reinforcement

$$V_{Rd,c} = \left[C_{Rd,c}\, k(100\rho_l f_{ck})^{1/3} + k_1\sigma_{cp}\right] b_w d \geq (v_{min} + k_1\sigma_{cp}) b_w d$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2{,}0 \qquad \rho_l = \frac{A_{sl}}{bd} \leq 0{,}02 \qquad v_{min} = 0.035 k^{3/2} f_{ck}^{1/2}$$

Beam	***f**$_{ck}$*	***d***	***C**$_{Rd,c}$*	***k***	***b***	***A**$_{sl}$*	***ρ**$_l$*	***v**$_{min}$*	***V**$_{Rd,c}$*	***v**$_{min}$**b**$_w$**d***
	MPa	m	-	-	m	cm^2	-	kPa	kN	kN
V_1										
V_2	30	0.364	0.12	1.74	0.25	3.77	0.00414	440.47	44.04	40.08
V_3										

Calculation of shear reinforcement

Maximum longitudinal spacing of the stirrups

$$s_{l,max} \leq 0{,}75d(1 + cot\alpha)$$

d	α	cotα	$s_{l,max}$
m	°	-	m
0.364	90	0	0.273

Minimum reinforcement

$$\rho_{w,min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yk}} \qquad \frac{A_{sw}}{s} = \rho_w b_w sin\alpha$$

f_{ck}	f_{yk}	$\rho_{w,min}$	α	b_w	$A_{sw,min}/s$	A_{sw}/s		
MPa	MPa	-	°	m	cm^2/m	cm^2/m	-	
30	500	0.00088	90	0.25	2.19	2.26	Ø6 // 0.25	2 legs

Shear reinforcement

$$\upsilon_1 = 0.6\left[1 - \frac{f_{ck}}{250}\right] \qquad V_{Rd,s} = \frac{A_{sw}}{s} z\, f_{ywd} \cot\theta \qquad V_{Rd,max} = \frac{\alpha_{cw} b_w z v_1 f_{cd}}{\cot\theta + \tan\theta}$$

f_{ck}	α_{cw}	b_w	d	z	v_1	f_{cd}	$\cot\Theta$	$V_{Rd,max}$
MPa	-	m	m	m	-	MPa	-	kN
30	1	0.25	0.364	0.328	0.528	20	2.5	298.23

	support	V_{Ed}	A_{sw}/s	Assumed reinforcement		
		kN	cm²/m	cm²/m	-	
V_1	edge	51.76	1.45	2.26	Ø6 // 0.25	2 legs
	internal	62.11	1.74			
V_2	internal	56.94	1.60			
	internal	56.94	1.60			
V_3	edge	51.76	1.45			
	internal	62.11	1.74			

Assumed shear reinforcement: Ø6 // 0.25

APPENDIX A.3

FLAT PLATE WITH SPANDREL BEAMS
7.2m x7.2m

Materials

Concrete	f_{ck}	f_{cm}	f_{ctm}	E_{cm}	ρ_c	γ_c	**Steel**	f_{yk}	E_s	γ_s
C30/37	MPa	MPa	MPa	GPa	kN/m^3	-	**A500**	MPa	GPa	-
	30	38	2.9	33	25.0	1.5		500	210	1.15

Actions

Category of loaded area: A - Areas for domestic and residential activities

Permanent actions	g_k (kN/m^2)
Imposed load	5.0
External walls	3.8

Variable actions	q_k (kN/m^2)
Imposed load	2.0

	p_d (kN/m^2)
total design load	**20.89**

Predimensioning of slab

concrete lightly stressed: ρ = 0,5%	***l***	***l/d***	***d***
	m	-	cm
Slab supported on columns without beams	7.2	24	30.0

l - span

d - effective depth of a cross-section

Dimensioning of slab

Area of slab

B_x	B_y	A
m	m	m^2
28.8	14.4	414.72

Geometry of slab

l/d	d_{slab}	h_{slab}
-	m	m
24	**0.30**	**0.33**

Dimensioning of columns

column on:	position of column	A_{inf}	β	c
		m²	-	m
ground, 1st floor	internal	51.84	1.15	0.55
	edge	25.92	1.40	0.45
	corner	12.96	1.50	0.35
2nd, 3rd	internal	51.84	1.15	0.45
	edge	25.92	1.40	0.40
	corner	12.96	1.50	0.30
4th, 5th, 6th	internal	51.84	1.15	0.35
	edge	25.92	1.40	0.30
	corner	12.96	1.50	0.30

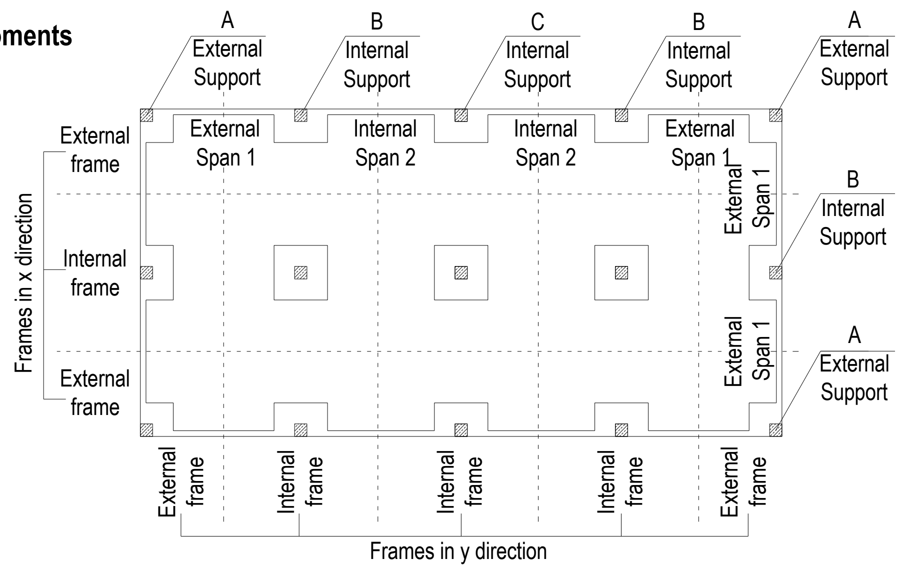
Reinforcement ratio assumed in columns: 2%

β - recommended values where adjacent spans do not differ more than 25%

Ultimate limit state

Bending

Bending moments



Direction x

strip	$M_{Ed,x,A}$ kNm	$M_{Ed,x,1}$ kNm	$M_{Ed,x,B}$ kNm	$M_{Ed,x,2}$ kNm	$M_{Ed,x,C}$ kNm
column	227.42	334.16	545.80	311.89	496.18
middle	97.46	222.78	233.91	207.92	212.65

Direction y

strip	$M_{Ed,A}$ kNm	$M_{Ed,1}$ kNm	$M_{Ed,B}$ kNm
column	227.42	334.16	606.45
middle	97.46	222.78	259.91

Height of a slab area

$\boldsymbol{\varnothing_{bottom,x}}$ mm	$\boldsymbol{\varnothing_{bottom,y}}$ mm	$d_{slab,x}$ mm	$d_{slab,y}$ mm	c_{nom} mm	h mm
16	12	**290**	**304**	20	330

Calculation of bottom reinforcement

Minimum reinforcement

$$A_{s,\min} = 0{,}26\frac{f_{ctm}}{f_{yk}}b_t d \geq 0{,}0013 b_t d$$

b mm	concrete cover mm	***d*** mm	$A_{s,min}$ cm²/m	$0.0013b_td$ cm²/m	$A_{s,min,bottom}$ -	cm²
1000	20	290	4.37	3.77	Ø8//0.125	4.02
1000	20	304	4.58	3.95	Ø8//0.125	4.02

Distance between bars

s ≥ 0.02m

$$s \geq máximo\{\phi ; d_g + 5mm ; 20mm\}$$

Reinforcement in direction x

$$\mu = \frac{M_{Ed}}{bd^2 f_{cd}} \qquad \omega = 0{,}973(1-\sqrt{1-2{,}056\mu}) \qquad A_s = \frac{\omega b d f_{cd}}{f_{yd}}$$

`

		M_{Ed}	μ	ω	$A_{s,min}$	$A_{s,bottom}$	
		kNm/m	-	-	cm²/m	-	cm²/m
$M_{Ed,1}$	column strip	334.16	0.055	0.057	7.59	Ø12//150	7.54
	middle strip	222.78	0.037	0.038	5.01	Ø10//150	5.24
$M_{Ed,2}$	column strip	311.89	0.052	0.053	7.07	Ø12//150	7.54
	middle strip	207.92	0.034	0.035	4.67	Ø10//150	5.24

Reinforcement in direction y

		M_{Ed}	μ	ω	$A_{s,min}$	$A_{s,bottom}$	
		kNm/m	-	-	cm²/m	-	cm²/m
$M_{Ed,1}$	column strip	334.16	0.050	0.052	7.22	Ø12//150	7.54
	middle strip	222.78	0.033	0.034	4.77	Ø10//150	5.24

Calculation of top reinforcement

Top reinforcement in direction x

$$\mu = \frac{M_{Ed}}{bd^2 f_{cd}} \qquad A_s = \frac{\omega b d f_{cd}}{f_{yd}} \qquad \omega = 0{,}973(1-\sqrt{1-2{,}056\mu})$$

		M_{Ed}	b	*μ*	*ω*	*A_s*	*$A_{s,top,x}$*	
		kNm	m	-	-	cm²/m	-	cm²/m
$M_{Sd,A}$	column strip	227.42	3.6	0.038	0.038	5.11	Ø10//125	6.28
	middle strip	97.46		0.016	0.016	2.17	Ø10//125	6.28
$M_{Sd,B}$	column strip	545.80		0.090	0.095	12.64	Ø16//150	13.40
	middle strip	233.91		0.039	0.039	5.26	Ø10//125	6.28
$M_{Sd,C}$	column strip	496.18		0.082	0.086	11.44	Ø16//150	13.40
	middle strip	212.65		0.035	0.036	4.77	Ø10//125	6.28

Top reinforcement in direction y

		M_{Ed}	b	μ	ω	A_s	$A_{s,top,y}$	
		kNm	m	-	-	cm²/m	-	cm²/m
$M_{Sd,A}$	column strip	227.42	3.6	0.034	0.035	5.36	Ø10//125	6.28
	middle strip	97.46		0.015	0.015	2.27	Ø10//125	6.28
$M_{Sd,B}$	column strip	606.45		0.091	0.096	13.25	Ø16//150	13.40
	middle strip	259.91		0.039	0.040	5.52	Ø10//125	6.28

Punching

Vertical force

column position		internal	edge	corner
V_{Ed}	kN	1082.94	633.81	363.07

Dimensions

h_{slab}	d
m	m
0.33	0.297

Punching shear resintance without shear reinforcement $v_{Rd,c}$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2,0$$

$$\rho_{lx} = \frac{A_{sl,x}}{b_y d} \leq 0,02$$

$$v_{Rd,c} = C_{Rd,c}\, k (100 \rho_l\, f_{ck})^{1/3} + k_1 \sigma_{cp} \geq v_{min} + k_1 \sigma_{cp}$$

$$\rho_l = \sqrt{\rho_{lx}\, \rho_{ly}} \leq 0,02$$

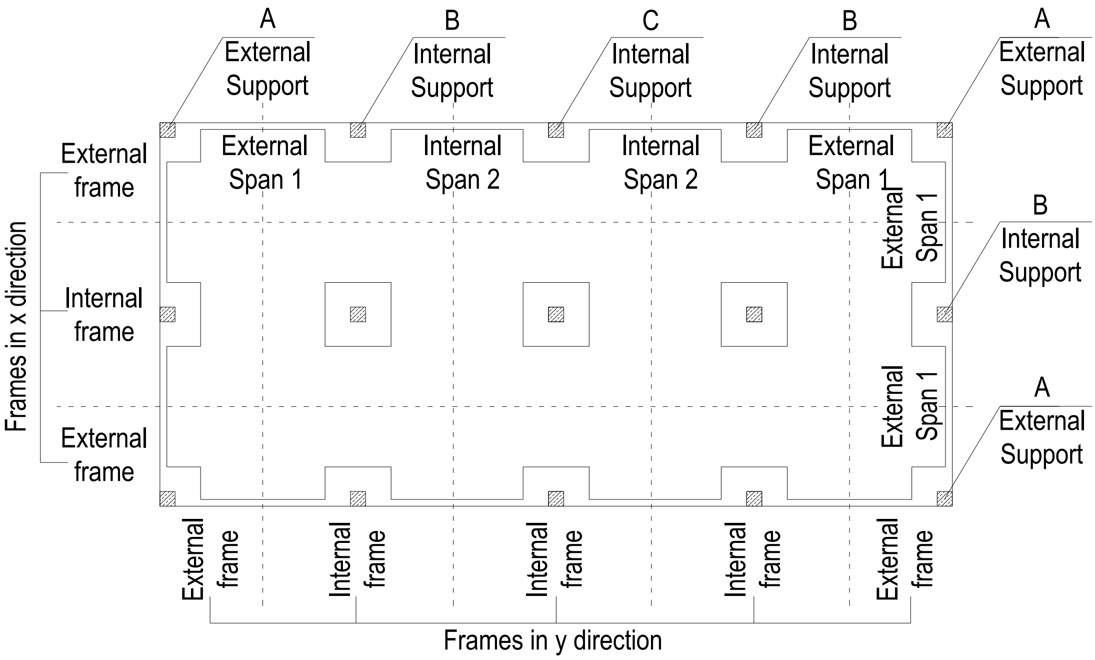
$$\rho_{ly} = \frac{A_{sl,y}}{b_x d} \leq 0,02$$

$$d_{eff} = \frac{d_x + d_y}{2}$$

$$v_{min} = 0,035 k^{3/2} f_{ck}^{\,1/2}$$

k_{slab} calculated	k_{slab} assumed	d_{slab}	$C_{Rd,c}$
-	-		-
1.821	1.821	0.297	0.120

Punching shear resintance without shear reinforcement in the slab



frame in direction x	support	position of column	$A_{s,top,x}$	$A_{s,top,y}$	$\rho_{l,slab}$	$v_{Rd,c}$	v_{min}
			cm^2/m	cm^2/m	-	kPa	kPa
exterior	A	corner	6.28	6.28	0.002116	404.49	470.93
	B	edge	13.40	6.28	0.003090	458.94	
	C	edge	13.40	6.28	0.003090	458.94	
interior	A	edge	6.28	13.40	0.003090	458.94	
	B	internal	13.40	13.40	0.004513	520.71	
	C	internal	13.40	13.40	0.004513	520.71	

Verification of necessity of use of shear punching reinforcement in the slab

storey	c	u_1	β	V_{Ed}	v_{Ed}	$v_{Rd,c}$	Necessity of using shear punching reinforcement
	m	*m*	-	*kN*	*kPa*	*kPa*	
Corner columns							
1st, 2nd	0.35	1.63	1.50	363.07	1122.87	470.93	
3rd, 4th	0.30	1.53	1.50	363.07	1196.12	470.93	necessary
5th, 6th	0.30	1.53	1.50	363.07	1196.12	470.93	
Edge columns							
1st, 2nd	0.45	3.22	1.40	633.81	928.97	470.93	
3rd, 4th	0.40	3.07	1.40	633.81	974.41	470.93	necessary
5th, 6th	0.30	2.77	1.40	633.81	1080.09	470.93	
Internal columns B							
1st, 2nd	0.55	5.93	1.15	1082.94	706.85	520.71	
3rd, 4th	0.45	5.53	1.15	1082.94	757.96	520.71	necessary
5th, 6th	0.35	5.13	1.15	1082.94	817.03	520.71	
Internal columns C							
1st, 2nd	0.55	5.93	1.15	1082.94	706.85	520.71	
3rd, 4th	0.45	5.53	1.15	1082.94	757.96	520.71	necessary
5th, 6th	0.35	5.13	1.15	1082.94	817.03	520.71	

$v_{Rd,c}$ - punching shear resintance without shear reinforcement

v_{Ed} - punching shear stress

u_1 - length of control perimeter in the slab

$$v_{Ed} = \frac{\beta V_{Ed}}{u_1 d_{eff,slab}}$$

Calculation of punching reinforcement

There is an assumption that: $v_{Ed} = v_{Rd,cs}$

$$f_{ywd,ef} = 250 + 0.25d \leq f_{ywd}$$

$$\frac{A_{sw}}{s_r} = \frac{v_{Rd,cs} - 0.75 v_{Rd,c}}{1.5 d\, f_{ywd,ef} \dfrac{1}{u_1\, d} sen\,\alpha}$$

Shear punching reinforcement in the slab

Corner columns

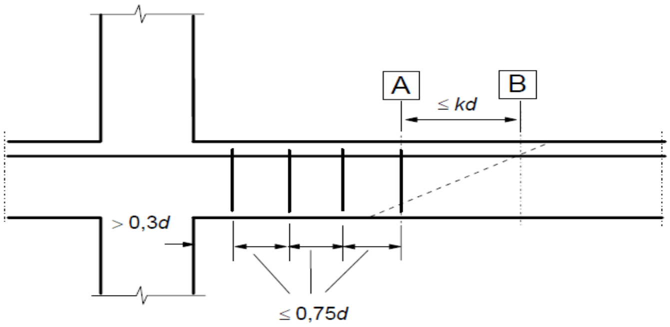
storey	u_2	$v_{Rd,cs}$	$v_{Rd,c}$	d	$f_{ywd,ef}$	α	A_{sw} / s_r
	m	kPa	kPa	m	MPa	°	cm^2/m
1st, 2nd	1.633	1122.9	470.93	0.297	324.25	90	25.84
3rd, 4th	1.533	1196.1	470.93	0.297	324.25	90	26.57
5th, 6th	1.533	1196.1	470.93	0.297	324.25	90	26.57

Edge columns

storey	u_1	$v_{Rd,cs}$	$v_{Rd,c}$	d	$f_{ywd,ef}$	α	A_{sw} / s_r
	m	kPa	kPa	m	MPa	°	cm^2/m
1st, 2nd	3.216	929.0	470.93	0.297	324.25	90	38.07
3rd, 4th	3.066	974.4	470.93	0.297	324.25	90	39.16
5th, 6th	2.766	1080.1	470.93	0.297	324.25	90	41.34

Internal columns

support	storey	V_{Ed}	u_2	$v_{Rd,cs}$	$v_{Rd,c}$	d	$f_{ywd,ef}$	α	A_{sw} / s_r
		kN	m	kPa	kPa	m	MPa	°	cm^2/m
internal B	1st, 2nd	1082.94	5.932	706.85	520.71	0.297	324.25	90	38.58
	3rd, 4th	1082.94	5.532	757.96	520.71	0.297	324.25	90	41.79
	5th, 6th	1082.94	5.132	817.03	520.71	0.297	324.25	90	45.00
internal C	1st, 2nd	1082.94	5.932	706.85	520.71	0.297	324.25	90	38.58
	3rd, 4th	1082.94	5.532	757.96	520.71	0.297	324.25	90	41.79
	5th, 6th	1082.94	5.132	817.03	520.71	0.297	324.25	90	45.00



	m
d	0.297
0.3d	0.089
0.5d	0.1485
0.75d	0.223
kd	0.446

The distance between the face of a support and the nearest shear reinforcement

The recommended value for *k* is 1.5.

Punching shear reinforcement should be placed between the column and *kd* inside the control perimeter at which shear reinforcement is no longer required.
The distance between the face of a support and the nearest shear reinforcement taken into account in the design should not exceed *d/2* (Eurocode 2, 2010)
For slabs with enlarged column heads where $l_H > 2h_H$ control sections both within the head and in the slab should be checked.

Shear punching reinforcement in the slab

The control perimeter at which shear reinforcement is not required

storey	u_{out} $u_{out,ef}$	r_{out}	c	s_r	A_{sw}	A_{sw}/s_r	$(A_{sw}/s_r)_{MIN}$
	m	m	m	m	cm^2	cm^2/m	cm^2/m
Corner columns							
1st, 2nd	3.89	2.26	0.35	0.200	5.65	28.27	25.84
3rd, 4th	3.89	2.29	0.30	0.200	5.65	28.27	26.57
5th, 6th	3.89	2.29	0.30	0.200	5.65	28.27	26.57
Edge columns							
1st, 2nd	6.34	1.88	0.45	0.200	10.18	50.89	38.07
3rd, 4th	6.34	1.89	0.40	0.200	10.18	50.89	39.16
5th, 6th	6.34	1.92	0.30	0.200	9.05	45.24	41.34
Internal columns B							
1st, 2nd	8.05	1.28	0.55	0.200	7.85	39.27	38.58
3rd, 4th	8.05	1.28	0.45	0.175	7.85	44.88	41.79
5th, 6th	8.05	1.28	0.35	0.150	7.85	52.36	45.00
Internal column C							
1st, 2nd	8.05	1.28	0.55	0.200	7.85	39.27	38.58
3rd, 4th	8.05	1.28	0.45	0.175	7.85	44.88	41.79
5th, 6th	8.05	1.28	0.35	0.150	7.85	52.36	45.00

storey	Verification of the distance* ≤ kd=0.446m	Verification of the spacing[1] ≤ 2d=0.594m	reinforcement per perimeter	Number of perimeters	$r_{assumed}$	$r_{obligatory}$
	m	m			m	m
Corner columns						
1st, 2nd	0.114	0.602	5#12	9	1.925	1.811
3rd, 4th	0.058	0.594	5#12	9	1.9	1.842
5th, 6th	0.058	0.594	5#12	9	1.9	1.842
Edge columns						
1st, 2nd	0.144	0.547	9#12	7	1.575	1.431
3rd, 4th	0.103	0.538	9#12	7	1.55	1.447
5th, 6th	0.022	0.585	8#12	7	1.5	1.478
Internal columns B						
1st, 2nd	0.189	0.531	12#10	4	1.025	0.836
3rd, 4th	0.064	0.556	10#10	4	0.9	0.836
5th, 6th	0.089	0.572	10#10	5	0.925	0.836
Internal column C						
1st, 2nd	0.189	0.531	12#10	4	1.025	0.836
3rd, 4th	0.064	0.556	10#10	4	0.9	0.836
5th, 6th	0.089	0.572	10#10	5	0.925	0.836

* - the distance between the outermost perimeter of shear reinforcement and the control perimeter u_{out}

[1] - the spacing of link legs around a perimeter should not exceed 2d

Edge beams

Minimum reinforcement

$$A_{s,\min} = 0{,}26\frac{f_{ctm}}{f_{yk}} b_t d \geq 0{,}0013 b_t d$$

b	concrete cover	***d***	***$A_{s,min}$***	*$0.0013b_td$*		***A_s***
cm	mm	cm	cm²/m	cm²/m	-	cm²
25	20	37	1.39	1.20	2 Ø12	2.26

Bottom reinforcement

	M_{Ed}	μ	ω	$A_{s,min}$		A_s
	kNm	-	-	cm²	-	cm²/m
V_1	33.19	0.049	0.050	2.13	2Ø12	2.26
V_2	26.55	0.039	0.040	1.69	2Ø12	2.26
V_3	33.19	0.049	0.050	2.13	2Ø12	2.26

Top reinforcement

		M_{Ed}	μ	ω	$A_{s,min}$	A_s	
		kNm	-	-	cm²	-	cm²/m
V_1	edge	31.87	0.047	0.048	2.04	2Ø12	2.26
	internal	55.76	0.082	0.086	3.65	2Ø16	4.02
V_2	internal	47.80	0.071	0.073	3.11	2Ø16	4.02
	internal	47.80	0.071	0.073	3.11	2Ø16	4.02
V_3	edge	31.87	0.047	0.048	2.04	2Ø12	2.26
	internal	55.76	0.082	0.086	3.65	2Ø16	4.02

Verification of shear force should be defined in proximity of column head.

	support	V_{Ed}
		kNm
V_1	edge	60.99
	internal	73.18
V_2	internal	67.09
	internal	67.09
V_3	edge	60.99
	internal	73.18

Members not requiring design shear reinforcement

$$V_{Rd,c} = \left[C_{Rd,c}\, k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp}\right] b_w d \geq (v_{\min} + k_1 \sigma_{cp}) b_w d$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2,0 \qquad \rho_l = \frac{A_{sl}}{bd} \leq 0,02 \qquad v_{\min} = 0.035 k^{3/2} f_{ck}^{1/2}$$

Beam	f_{ck}	d	$C_{Rd,c}$	k	b	A_{sl}	ρ_l	v_{min}	$V_{Rd,c}$	$v_{min} b_w d$
	MPa	m	-	-	m	cm^2	-	kPa	kN	kN
V_1										
V_2	30	0.368	0.12	1.74	0.25	3.77	0.0041	438.94	44.26	40.38
V_3										

Calculation of shear reinforcement

Maximum longitudinal spacing of the stirrups

$$s_{l,max} \leq 0,75 d (1 + cot\alpha)$$

d	α	cotα	$s_{l,max}$
m	°	-	m
0.368	90	0	0.276

Minimum reinforcement

$$\rho_{w,\min} = \frac{0,08\sqrt{f_{ck}}}{f_{yk}} \qquad \frac{A_{sw}}{s} = \rho_w b_w sin\alpha$$

f_{ck}	f_{yk}	$\rho_{w,min}$	α	b_w	$A_{sw,min}/s$	A_{sw}/s	
MPa	MPa	-	°	m	cm^2/m	cm^2/m	-
30	500	0.00088	90	0.25	2.19	2.26	Ø6 // 0.25 2 legs

Shear reinforcement

$$\upsilon_1 = 0.6\left[1 - \frac{f_{ck}}{250}\right] \qquad V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} \cot\theta \qquad V_{Rd,max} = \frac{\alpha_{cw} b_w z v_1 f_{cd}}{\cot\theta + \tan\theta}$$

f_{ck}	α_{cw}	b_w	d	z	v_1	f_{cd}	$\cot\Theta$	$V_{Rd,max}$
MPa	-	m	m	m	-	MPa	-	kN
30	1	0.25	0.368	0.331	0.528	20	2.5	301.51

Calculation of shear reinforcement

	support	V_{Ed}	A_{sw}/s	Assumed reinforcement		
		kN	cm²/m	cm²/m	-	
V_1	edge	60.99	1.69	2.26	Ø6 // 0.25	2 legs
	internal	73.18	2.03			
V_2	internal	67.09	1.86			
	internal	67.09	1.86			
V_3	edge	60.99	1.69			
	internal	73.18	2.03			

Assumed shear reinforcement: Ø6 // 0.25

APPENDIX A.4

TWO-WAY SLAB WITH BEAMS

7.2m x 7.2m

Materials

Concrete	f_{ck}	f_{cm}	f_{ctm}	E_{cm}	ρ_c	γ_c	**Steel**	f_{yk}	E_s	γ_s
C30/37	MPa	MPa	MPa	GPa	kN/m³	-	**A500**	MPa	GPa	-
	30	38	2.9	33	25.0	1.5		500	210	1.15

Actions

Category of loaded area: A - Areas for domestic and residential activities

Permanent actions	g_k kN/m²
Imposed load	5.0
External walls	3.8

Variable actions	q_k kN/m²
Imposed load	2.0

Pre-dimensioning of slab

concrete lightly stressed: ρ = 0,5%	***l***	***l/d***	***d***
	m	-	cm
end span of two-way spanning slab continouos over one long side	7.2	26	27.7
interior span of two-way spanning slab	7.2	30	24.0

l - span

d - effective depth of a cross-section

Height of slab

l_x	l_y	l/d	$Ø_{stirrup}$	$Ø_{bottom}$	$Ø_{top}$	d	c_{nom}	h
m	m	-	mm	mm	mm	mm	mm	mm
7.2	7.2	25.17	0	8	8	286	20	310

Area of slab

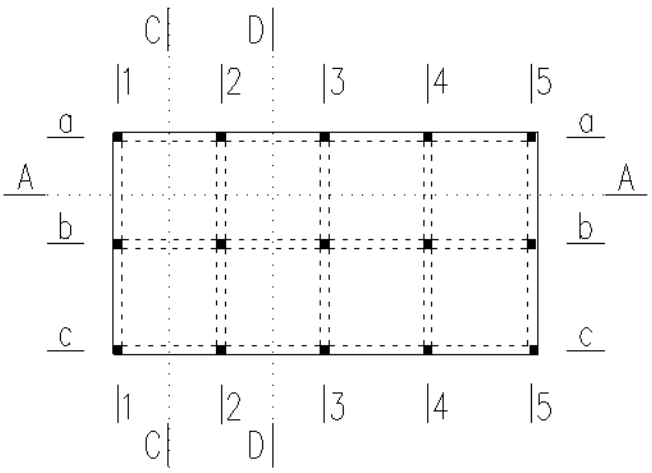
B_x	B_y	A
m	m	m^2
28.8	14.4	414.72

Predimensioning of columns

column on:	position of column	A_{inf}	β	c
		m^2	-	m
ground, 1st floor	internal	51.84	1.15	0.60
	edge	25.92	1.40	0.50
	corner	12.96	1.50	0.40
2nd, 3rd	internal	51.84	1.15	0.45
	edge	25.92	1.40	0.40
	corner	12.96	1.50	0.40
4th, 5th, 6th	internal	51.84	1.15	0.40
	edge	25.92	1.40	0.40
	corner	12.96	1.50	0.40

β - recommended values where adjacent spans do not differ more than 25%

Bending moments in slab



$$\mu = \frac{M_{Ed}}{bd^2 f_{cd}}$$

$$\omega = 0{,}973(1 - \sqrt{1 - 2{,}056\mu})$$

$$A_s = \frac{\omega\, b\, d\, f_{cd}}{f_{yd}}$$

	β	M_{Ed}	μ	ω	A_s
	-	kNm/m	-	-	cm²/m
Two adjacent edges discontinuous					
Negative moment at continuous edge	0.047	49.97	0.0305	0.0310	4.08
Positive moment at mid-span	0.036	38.28	0.0234	0.0237	3.12
One edge discontinuous					
Negative moment at continuous edge	0.039	41.47	0.0253	0.0257	3.38
Positive moment at mid-span	0.030	31.90	0.0195	0.0197	2.59
β - bending moment coefficient from BS 8110 : Part 1 : 1997					

Moments in direction of x

Cross section	$M_{Ed,1-1}$	$M_{Ed,1-2}$	$M_{Ed,2-2}$	$M_{Ed,2-3}$	$M_{Ed,3-3}$
	support	mid-span	support	mid-span	support
	kNm/m	kNm/m	kNm/m	kNm/m	kNm/m
A-A	0.00	38.28	-45.72	31.90	-41.47

Moments in direction of y

Cross section	$M_{Ed,a-a}$	$M_{Ed,a-b}$	$M_{Ed,b-b}$
	support	mid-span	support
	kN/m	kN/m	kN/m
C-C	0.00	38.28	-45.72
D-D	0.00	31.90	41.47

Ultimate limit state

Bending

Minimal reinforcement in slab

$$A_{s,\min} = 0,26\frac{f_{ctm}}{f_{yk}}b_t d \geq 0,0013 b_t d$$

b	concrete cover	d	$A_{s,min}$	$0.0013b_td$	Ø10//175
mm	mm	mm	cm²/m	cm²/m	cm²/m
1000	20	286	4.31	3.72	4.49

Distance among pararel bars

$$s \geq máximo\{\phi; d_g + 5mm; 20mm\}$$

s = 20mm

Reinforcement in direction of x

Cross section A-A

$$\mu = \frac{M_{Ed}}{bd^2 f_{cd}}$$

$$\omega = 0{,}973(1 - \sqrt{1 - 2{,}056\mu})$$

$$A_s = \frac{\omega b\, d\, f_{cd}}{f_{yd}}$$

	M_{Ed}	μ	ω	$A_{s,min}$	$A_{s,assumed}$	
	kNm/m	-	-	cm^2/m		cm^2/m
$M_{Ed,1-1}$	0.000	0.000	0.000	0.00		
$M_{Ed,1-2}$	38.277	0.023	0.024	3.12		
$M_{Ed,2-2}$	-45.719	0.028	0.028	3.73	Ø10//175	4.49
$M_{Ed,2-3}$	31.897	0.019	0.020	2.59		
$M_{Ed,3-3}$	-41.466	0.025	0.026	3.38		

Reinforcement in direction of y

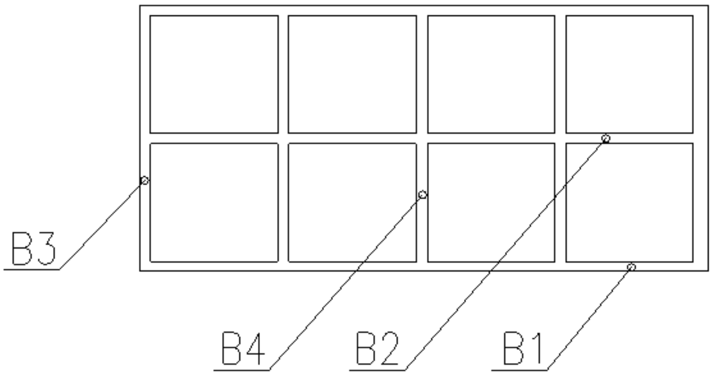
Cross section C-C

	M_{Ed}	μ	ω	$A_{s,min}$	$A_{s,assumed}$	
	kNm/m	-	-	cm^2/m		cm^2/m
$M_{Ed,a-a}$	0.000	0.000	0.000	0.00		
$M_{Ed,a-b}$	38.277	0.023	0.024	3.12	Ø10//175	4.49
$M_{Ed,b-b}$	-45.719	0.028	0.028	3.73		

Cross section D-D

	M_{Ed}	μ	ω	$A_{s,min}$	$A_{s,assumed}$	
	kNm/m	-	-	cm^2/m	cm2/m	
$M_{Ed,a-a}$	0.000	0.000	0.000	0.00		
$M_{Ed,a-b}$	31.897	0.019	0.020	2.59	Ø10//175	4.49
$M_{Ed,b-b}$	41.466	0.025	0.026	3.38		

Bending moments in beams

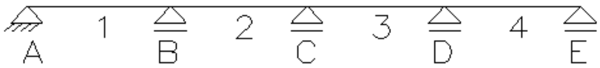


Load

g_k	12.75	kN/m^2
q_k	2	kN/m^2
beam design dead load	3.80	kN/m
wall design dead load	13.08	kN/m

Edge beam B1

External beam B1 is a 4-span beam.

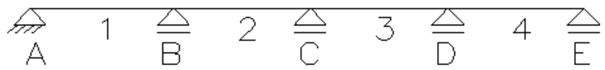


		loading coeficients		Partial bending moments				
		normal loading	triangle loading	wall	1.0*g_k	1.5q_k + 0.35gk	M_{Ed}	d
		-	-	kNm	kNm	kNm	kNm	m
M1	min	0.000	-0.015	0.00	-35.69	-20.89	-56.58	0.177
	max	0.100	0.067	87.50	159.42	93.31	340.23	0.435
MB	min	-0.121	-0.098	-105.87	-232.23	-135.93	-474.03	0.513
	max	0.013	0.009	11.37	21.42	12.53	45.32	0.159
M2	min	0.000	-0.028	0.00	-66.62	-39.00	-105.62	0.242
	max	0.080	0.056	70.00	133.25	77.99	281.24	0.395
MC	min	-0.107	-0.067	-93.62	-159.42	-93.31	-346.36	0.439
	max	0.036	0.023	31.50	54.73	32.03	118.26	0.256

d_{min}	0.513

Internal beam B2

Internal beam B2 is a 4-span beam

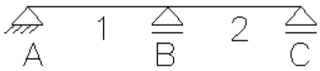


		loading coeficients		Partial bending moments				
		normal loading	triangle loading	beam	$1.0*g_k$	$1.5q_k$ + 0.35gk	M_{Ed}	d
		-	-	kNm	kNm	kNm	kNm	m
M1	min	0.000	-0.015	0.00	-71.38	-41.78	-113.16	0.217
	max	0.100	0.067	19.68	318.85	186.62	525.15	0.468
MB	min	-0.121	-0.098	-23.82	-464.47	-271.85	-760.14	0.563
	max	0.013	0.009	2.56	42.83	25.07	70.46	0.171
M2	min	0.000	-0.028	0.00	-133.25	-77.99	-211.24	0.297
	max	0.080	0.056	15.75	266.50	155.98	438.23	0.427
MC	min	-0.107	-0.067	-21.06	-318.85	-186.62	-526.53	0.468
	max	0.036	0.023	7.09	109.45	64.06	180.60	0.274

d_{min} 0.563

Edge beam B3

External beam B3 is a 2-span beam.

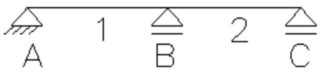


		loading coeficients		Partial bending moments				
		normal loading	triangle loading	wall +beam	$1.0*g_k$	$1.5q_k$ + 0.35gk	M_{Ed}	d
		-	-	kNm	kNm	kNm	kNm	m
M1	min	0	-0.018	0.00	-42.83	-25.07	-67.90	0.194
	max	0.096	0.065	84.00	154.66	90.52	329.19	0.428
MB	min	-0.125	-0.078	-109.37	-185.60	-108.63	-403.60	0.474
	max	-0.063	-0.039	-55.12	-92.80	-54.31	-202.24	0.335

d_{min} 0.474

Internal beam B4

Internal beam B4 is a 2-span beam.



		loading coeficients		Partial bending moments				
		normal loading	triangle loading	beam	1.0^*g_k	$1.5q_k + 0.35gk$	M_{Ed}	d
		-	-	kNm	kNm	kNm	kNm	m
M1	min	0	-0.018	0.00	-85.66	-50.14	-135.80	0.238
	max	0.096	0.065	18.90	309.33	181.05	509.27	0.461
MB	min	-0.125	-0.078	-24.60	-371.20	-217.26	-613.06	0.505
	max	-0.063	-0.039	-12.40	-185.60	-108.63	-306.63	0.357

d_{min}	0.505

Dimensions of beams

	h	b	A_c	d
	m	m		m
B1	0.60	0.30	0.0018	0.560
B2	0.60	0.40	0.0024	0.560
B3	0.60	0.30	0.0018	0.560
B4	0.60	0.40	0.0024	0.560

Calculated reinforcement

Edge beam B1

Bottom reinforcement

	M_{Ed}	μ	ω	$A_{s,min}$	A_s	
	kNm	-	-	cm^2	-	cm^2
M1	340.23	0.181	0.202	15.61	2Ø25+2Ø20	16.10
M2	281.24	0.150	0.164	12.62	2Ø25+2Ø16	13.84

Top reinforcement

	M_{Ed}	μ	ω	$A_{s,min}$	A_s	
	kNm	-	-	cm^2	-	cm^2
MB	-474.03	0.252	0.298	23.02	5Ø25	24.54
MC	-346.36	0.184	0.206	15.93	2Ø25+2Ø20	16.10

Internal beam B2

Bottom reinforcement

	M_{Ed}	μ	ω	$A_{s,min}$	A_s	
	kNm	-	-	cm^2	-	cm^2
M1	525.15	0.280	0.339	26.14	6Ø25	29.45
M2	438.23	0.233	0.271	20.94	5Ø25	24.54

Top reinforcement

	M_{Ed}	μ	ω	$A_{s,min}$	A_s	
	kNm	-	-	cm^2	-	cm^2
MB	-760.14	0.405	0.574	27.74	5Ø25	24.54
MC	-526.53	0.280	0.340	16.41	2Ø25+2Ø20	16.10

Edge beam B3

Bottom reinforcement

	M_{Ed}	μ	ω	$A_{s,min}$	A_s	
	kNm	-	-	cm^2	-	cm^2
M1	329.19	0.175	0.195	15.04	2Ø25+2Ø20	16.10

Top reinforcement

	M_{Ed}	μ	ω	$A_{s,min}$	A_s	
	kNm	-	-	cm^2	-	cm^2
MB	-403.60	0.215	0.246	19.00	4Ø25	19.63

Internal beam B4

Bottom reinforcement

	M_{Ed}	μ	ω	$A_{s,min}$	A_s	
	kNm	-	-	cm^2	-	cm^2
M1	509.27	0.271	0.326	25.15	6Ø25	29.45

Top reinforcement

	M_{Ed}	μ	ω	$A_{s,min}$	A_s	
	kNm	-	-	cm^2	-	cm^2
MB	-613.06	0.326	0.415	20.04	5Ø25	24.54

Shear

Shear forces in beam

Edge beam B1

		type of loading		
		normal	triangle	V_{Ed}
		-	-	kN
VA	min	-0.053	-0.033	-18.57
				0.00
V1B	min	-0.621	-0.326	-195.24
	max	0.014	0.009	5.01
V2B	min	-0.072	-0.045	-25.28
	max	0.603	0.314	188.65
V2C	min	-0.576	-0.298	-179.49
	max	0.107	0.067	37.62
V3C	min	-0.091	-0.057	-32.00
	max	0.591	0.307	184.62

coefficients taken from tables in [1]

Internal beam B2

		type of loading		
		normal	triangl e	V_{Ed}
		-	-	kN
VA	min	-0.053	-0.033	-30.69
				0.00
V1B	min	-0.621	-0.326	-315.02
	max	0.014	0.009	8.31
V2B	min	-0.072	-0.045	-41.82
	max	0.603	0.314	304.02
V2C	min	-0.576	-0.298	-288.98
	max	0.107	0.067	62.24
V3C	min	-0.091	-0.057	-52.94
	max	0.591	0.307	297.41

coefficients taken from tables in [1]

Edge beam B3

		type of loading		
		normal	triangle	V_{Ed}
		-	-	kN
VA	min	-0.05	-0.032	-17.83
	max	0.45	0.219	135.15
V1B	min	-0.617	-0.323	-193.66
	max	0.017	0.011	6.11
V2B	min	-0.083	-0.053	-29.56
	max	0.583	0.303	182.18

Internal beam B4

		type of loading		
		normal	triangle	V_{Ed}
		-	-	kN
VA	min	-0.05	-0.032	-29.59
	max	0.45	0.219	215.61
V1B	min	-0.617	-0.323	-312.33
	max	0.017	0.011	10.15
V2B	min	-0.083	-0.053	-49.03
	max	0.583	0.303	293.50

Members not requiring design shear reinforcement

$$V_{Rd,c} = \left[C_{Rd,c}\, k(100\rho_l f_{ck})^{1/3} + k_1\sigma_{cp}\right] b_w d \geq (v_{min} + k_1\sigma_{cp}) b_w d$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2{,}0 \qquad \rho_l = \frac{A_{sl}}{bd} \leq 0{,}02 \qquad v_{min} = 0.035 k^{3/2} f_{ck}^{1/2}$$

Beam	f_{ck}	d	$C_{Rd,c}$	k	b_w	A_{sl}	ρ_l	v_{min}	$V_{Rd,c}$	$V_{Rd,c\ min}$
	MPa	m	-	-	m	cm2		kPa	kN	kN
B1	30	0.560	0.12	19.907	0.30	3.77	2.2460	538.43	7.57	0.09
B2		0.560		19.907	0.40		1.6845	538.43	9.17	0.12
B3		0.560		19.907	0.30		2.2460	538.43	7.57	0.09
B4		0.560		19.907	0.40		1.6845	538.43	9.17	0.12

Members with vertical shear reinforcement

$$V_{Rd,max} = \frac{\alpha_{cw} b_w z v_1 f_{cd}}{\cot\theta + \tan\theta}$$

Shear resistance

Beam	f_{ywd}	α_{cw}	b_w	d	z	v_1	f_{cd}	cotΘ	$V_{Rd,s}$	$V_{Rd,max}$
	MPa	-	m	m	m	-	MPa	-	kN	kN
B1	434.78	1	0.30	0.56	0.5036	0.528	20	2.5	2201	550.08
B2	434.78	1	0.40	0.56	0.5036	0.528	20	2.5	2201	733.45
B3	434.78	1	0.30	0.56	0.5036	0.528	20	2.5	2201	550.08
B4	434.78	1	0.40	0.56	0.5036	0.528	20	2.5	2201	733.45

Calculation of shear reinforcement

Maximum longitudinal spacing of the stirrups

$$s_{l,max} \leq 0{,}75d(1 + \cot\alpha)$$

d	α	cotα	$s_{l,max}$
m	°	-	m
0.42	90	0	0.315

Minimal reinforcement

$$\rho_{w,min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yk}} \qquad \frac{A_{sw}}{s} = \rho_w b_w sin\alpha$$

f_{ck}	f_{yk}	$\rho_{w,min}$	α	b_w	$A_{sw,min}/s$		A_{sw}/s	
MPa	MPa	-	°	m	cm²/m	cm²/m	-	
30	500	0.0009	90	0.30	2.63	4.02	Ø8 // 0.25	2 legs
30	500	0.0009	91	0.40	3.51	3.77	Ø6 // 0.30	4 legs

Shear reinforcement

$$\upsilon_1 = 0.6\left[1 - \frac{f_{ck}}{250}\right] \qquad V_{Rd,s} = \frac{A_{sw}}{s} z\, f_{ywd} \cot\theta \qquad V_{Rd,max} = \frac{\alpha_{cw} b_w z \nu_1 f_{cd}}{\cot\theta + \tan\theta}$$

f_{ck}	α_{cw}	b_w	d	z	v_1	f_{cd}	$cot\Theta$	$V_{Rd,max}$
MPa	-	m	m	m	-	MPa	-	kN
30	1	0.30	0.56	0.504	0.528	20	2.5	550.08

Edge beam B1

support	V_{Ed}	A_{sw}/s	Assumed reinforcement		
	kN	cm²/m	cm²/m	-	
V1B	-195.24	3.57	3.77	Ø6 // 0.30	4 legs
V2B	188.65	3.45	3.77	Ø6 // 0.30	4 legs
V2C	-179.49	3.28	3.77	Ø6 // 0.30	4 legs
V3C	184.62	3.37	3.77	Ø6 // 0.30	4 legs

Internal beam B2

support	V_{Ed}	A_{sw}/s	Assumed reinforcement		
	kN	cm²/m	cm²/m	-	
V1B	-315.02	5.76	8.04	Ø8 // 0.25	4 legs
V2B	304.02	5.55	8.04	Ø8 // 0.25	4 legs
V2C	-288.98	5.28	8.04	Ø8 // 0.25	4 legs
V3C	297.41	5.43	8.04	Ø8 // 0.25	4 legs

Edge beam B3

support	V_{Ed}	A_{sw}/s	Assumed reinforcement		
	kN	cm^2/m	cm^2/m	-	
VA	135.15	2.47	3.77	Ø6 // 0.30	4 legs
V1B	-193.66	3.54	3.77	Ø6 // 0.30	4 legs
V2B	182.18	3.33	3.77	Ø6 // 0.30	4 legs

Edge beam B4

support	V_{Ed}	A_{sw}/s	Assumed reinforcement		
	kN	cm^2/m	cm^2/m	-	
VA	215.61	3.94	4.02	Ø8 // 0.25	2 legs
V1B	-312.33	5.71	8.04	Ø8 // 0.25	4 legs
V2B	293.50	5.36	8.04	Ø8 // 0.25	4 legs

APPENDIX B

Table B.1: Slab with column heads – span 7.20m

	unit	quantity	price per unit	price
Concrete C30/37				
Column heads	m^3	18.43	77.50	1428.48
Slab with normal height	m^3	73.73	77.50	5713.92
Edge beams	m^3	4.86	77.50	376.65
Steel A500				
Ø6	kg	95.56	0.85	81.23
Ø8	kg	0.00	0.83	0.00
Ø10	kg	1707.09	0.80	1365.67
Ø12	kg	6035.32	0.78	4707.55
Ø16	kg	3448.66	0.77	2655.47
Ø20	kg	0.00	0.77	0.00
Formwork				
Column heads	m^2	61.44	11.00	675.84
Slab with normal height	m^2	368.64	11.00	4055.04
Edge beams	m^2	61.2	11.00	673.20
Labour				
Labor force - concrete pouring				
	euro/m3	97.02	4.50	436.59
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	11286.64	0.17	1918.73
			Total:	**24088.38€**

Table B.2: Slab with column heads – span 8.00m

	unit	quantity	price per unit	price
Concrete C30/37				
Column heads	m^3	23.33	77.50	1807.92
Slab with normal height	m^3	90.74	77.50	7032.04
Edge beams	m^3	5.36	77.50	415.01
Steel A500				
Ø6	kg	106.18	0.85	90.26
Ø8	kg	0.00	0.83	0.00
Ø10	kg	2096.43	0.80	1677.14
Ø12	kg	6499.06	0.78	5069.26
Ø16	kg	116.17	0.77	89.45
Ø20	kg	5099.09	0.77	3926.30
Formwork				
Column heads	m^2	75.60	11.00	831.60
Slab with normal height	m^2	453.68	11.00	4990.48
Edge beams	m^2	68	11.00	748.00
Labour				
Labor force - concrete pouring				
	euro/m3	119.42	4.50	537.39
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	13916.93	0.17	2365.88
			Total:	**29580.73€**

Table B.3: Slab with column heads – span 8.80m

	unit	quantity	price per unit	price
Concrete C30/37				
Column heads	m^3	28.80	77.50	2232.00
Slab with normal height	m^3	109.50	77.50	8486.56
Edge beams	m^3	5.85	77.50	453.38
Steel A500				
Ø6	kg	116.80	0.85	99.28
Ø10	kg	2525.69	0.80	2020.55
Ø12	kg	5026.56	0.78	3920.72
Ø16	kg	6402.29	0.77	4929.77
Ø20	kg	7661.94	0.77	5899.69
Formwork				
Column heads	m^2	93.12	11.00	1024.32
Slab with normal height	m^2	547.52	11.00	6022.72
Edge beams	m^2	80.96	11.00	890.56
Labour				
Labor force - concrete pouring				
	euro/m3	144.15	4.50	648.69
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	21733.29	0.17	3694.66
			Total:	**40322.90€**

Table B.4: Waffle slab - span 7.20m

	unit	quantity	price per unit	price
Concrete C30/37				
Column heads	m^3	32.77	77.50	2539.52
Slab with normal height	m^3	107.52	66.56	77.50
Edge beams	m^3	8.10	77.50	627.75
Steel A500				
Ø6	kg	119.46	0.85	101.54
Ø8	kg	0.00	0.83	0.00
Ø10	kg	565.87	0.80	452.70
Ø12	kg	1150.37	0.78	897.29
Ø16	kg	2292.31	0.77	1765.08
Ø20	kg	296.94	0.77	228.65
Ø25	kg			
Formwork				
Slab	m^2	468.72	11.00	5155.92
Molds returnable 0,8x0,8m^2				
	m^2	332.8	4.75	1580.80
Labour				
Labor force - concrete pouring				
	euro/m^3	107.43	4.50	483.43
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	4424.94	0.17	752.24
			Total:	**19743.30€**

Table B.5: Waffle slab - span 8.00m

	unit	quantity	price per unit	price
Concrete C30/37				
Column heads	m^3	32.77	77.50	2539.52
Slab with normal height	m^3	107.52	86.02	77.50
Edge beams	m^3	9.00	77.50	697.50
Steel A500				
Ø6	kg	1242.50	0.85	1056.12
Ø8	kg	802.37	0.83	665.97
Ø10	kg	1179.80	0.80	943.84
Ø12	kg	392.06	0.78	305.81
Ø16	kg	4007.47	0.77	3085.75
Ø20	kg	395.08	0.77	304.21
Ø25	kg			
Formwork				
Slab	m^2	572.00	11.00	6292.00
Molds returnable 0,8x0,8m^2				
	m^2	537.6	430.08	4.75
Labour				
Labor force - concrete pouring				
	euro/m^3	127.78	4.50	575.03
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	8019.28	0.17	1363.28
			Total:	**26538.15€**

Table B.6: Waffle slab - span 8.80m

	unit	quantity	price per unit	price
Concrete C30/37				
Column heads	m^3	32.77	77.50	2539.52
Slab with normal height	m^3	107.52	107.52	77.50
Edge beams	m^3	9.90	77.50	767.25
Steel A500				
Ø6	kg	1526.39	0.85	1297.43
Ø8	kg	395.38	0.83	328.16
Ø10	kg	1697.11	0.80	1357.69
Ø12	kg	790.65	0.78	616.71
Ø16	kg	3573.37	0.77	2751.49
Ø20	kg	500.12	0.77	385.09
Ø25	kg			
Formwork				
Slab	m^2	685.52	11.00	7540.72
Molds returnable 0,8x0,8m^2				
	m^2	537.6	4.75	2553.60
Labour				
Labor force - concrete pouring				
	euro/m^3	150.19	4.50	675.85
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	8483.01	0.17	1442.11
			Total:	**30850.42€**

Table B.7: Flat plate - span 5.60m

	unit	quantity	price per unit	price
Concrete C30/37				
Slab	m^3	65.23	76.50	4990.00
Edge beams	m^3	5.60	77.50	434.00
Steel A500				
Ø6	kg	74.33	0.85	63.18
Ø10	kg	891.93	0.80	713.55
Ø12	kg	554.06	0.78	432.17
Ø16	kg	1766.88	0.77	1360.50
Formwork				
Slab	m^2	250.88	11.00	2759.68
Edge beams	m^2	29.12	11.00	320.32
Labour				
Labor force - concrete pouring				
	euro/m^3	70.83	4.50	318.73
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	3287.20	0.17	558.82
			Total:	**11950.95€**

Table B.8: Flat plate - span 6.40m

	unit	quantity	price per unit	price
Concrete C30/37				
Slab	m^3	98.30	76.50	7520.26
Edge beams	m^3	5.60	6.40	77.50
Steel A500				
Ø6	kg	84.95	0.85	72.20
Ø10	kg	2729.37	0.80	2183.50
Ø12	kg	1847.25	0.78	1440.85
Ø16	kg	1429.61	0.77	1100.80
Formwork				
Slab	m^2	327.68	11.00	3604.48
Edge beams	m^2	29.12	33.28	11.00
Labour				
Labor force - concrete pouring				
	euro/m^3	104.70	4.50	471.17
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	6091.18	0.17	1035.50
			Total:	**18290.84€**

Table B.9: Flat plate - span 7.20m

	unit	quantity	price per unit	price
Concrete C30/37				
Slab	m^3	136.86	76.50	10469.61
Edge beams	m^3	5.60	7.20	77.50
Steel A500				
Ø6	kg	95.56	0.85	81.23
Ø8	kg	0.00	0.83	0.00
Ø10	kg	3430.82	0.80	2744.66
Ø12	kg	2180.74	0.78	1700.98
Ø16	kg	1894.66	0.77	1458.89
Ø20	kg	0.00	0.77	0.00
Formwork				
Slab	m^2	414.72	11.00	4561.92
Edge beams	m^2	37.44	11.00	411.84
Labour				
Labor force - concrete pouring				
	euro/m^3	144.06	4.50	648.26
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	7601.79	0.17	1292.30
			Total:	**23927.68€**

Table B.10: Slab with beams – span 7.20m

	unit	quantity	price per unit	price
Concrete C30/37				
Slab	m^3	128.56	77.50	9963.65
Beams	m^3	5.85	77.50	453.10
Steel A500				
Ø6	kg	281.30	0.85	239.10
Ø8	kg	7700.26	0.83	6391.22
Ø12	kg	0.00	0.78	0.00
Ø16	kg	200.03	0.77	154.02
Ø20	kg	585.96	0.77	451.19
Ø25	kg	4303.12	0.77	2514.00
Formwork				
Slab	m^2	414.72	11.00	4561.92
Beams	m^2	53.568	11.00	589.25
Labour				
Labor force - concrete pouring				
	euro/m^3	134.41	4.50	604.84
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	8767.54	0.17	1490.48
			Total:	**27412.76€**

Table B.11: Slab with beams – span 6.40m

	unit	quantity	price per unit	price
Concrete C30/37				
Slab	m^3	91.75	77.50	7110.66
Beams	m^3	3.94	77.50	305.54
Steel A500				
Ø6	kg	250.04	0.85	212.53
Ø8	kg	4237.18	0.83	3516.86
Ø16	kg	350.06	0.77	269.54
Ø20	kg	0.00	0.77	0.00
Ø25	kg	4455.72	0.77	2514.00
Formwork				
Slab	m^2	327.68	11.00	3604.48
Beams	m^2	53.568	40.448	11.00
Labour				
Labor force - concrete pouring				
	euro/m^3	95.69	4.50	430.62
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	4837.28	0.17	822.34
			Total:	**19231.49€**

Table B.12: Slab with beams – span 5.60m

	unit	quantity	price per unit	price
Concrete C30/37				
Slab	m^3	62.72	77.50	4860.80
Beams	m^3	3.94	2.74	77.50
Steel A500				
Ø6	kg	0.00	0.85	0.00
Ø8	kg	3487.15	0.83	2894.34
Ø10	kg	0.00	0.80	0.00
Ø12	kg	0.00	0.78	0.00
Ø16	kg	136.13	0.77	104.82
Ø20	kg	1336.85	0.77	1029.38
Ø25	kg	1614.10	0.77	2514.00
Formwork				
Slab	m^2	250.88	11.00	2759.68
Beams	m^2	31.64	11.00	348.04
Labour				
Labor force - concrete pouring				
	euro/m^3	65.46	4.50	294.59
Labor force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	4960.14	0.17	843.22
			Total:	**15861.53€**

APPENDIX C

Table C.1: Cost of columns for the slabs with column heads – span 7.20m

Concrete C30/37				
	m^3	57.02	77.50	4419.36
Steel A500				
	kg	8553.60	0.78	6671.81
Labour				
Labour force - concrete pouring				
	euro/m3	57.02	4.50	256.61
Labour force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	8553.60	0.17	1454.11
Formwork				
	m^2	252.00	11.00	2772.00
			Total:	**15573.89€**

Table C.2: Cost of columns for the slabs with column heads – span 8.00m

Concrete C30/37				
	m^3	65.05	77.50	5041.53
Steel A500				
	kg	9757.80	0.78	7611.08
Labour				
Labour force - concrete pouring				
	euro/m3	65.05	4.50	292.73
Labour force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	9757.80	0.17	1658.83
Formwork				
	m^2	267.84	11.00	2946.24
			Total:	**17550.41€**

Table C.3: Cost of columns for the slabs with column heads – span 8.80m

Concrete C30/37				
	m^3	76.62	77.50	5937.82
Steel A500				
	kg	11492.55	0.78	8964.19
Labour				
Labour force - concrete pouring				
	euro/m3	76.62	4.50	344.78
Labour force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	11492.55	0.17	1953.73
Formwork				
	m^2	291.60	11.00	3207.60
			Total:	**20408.12€**

Table C.4: Cost of columns for the slabs with beams – span 5.60m

Concrete C30/37				
	m^3	62.32	77.50	4829.49
Steel A500				
	kg	9347.40	0.85	7945.29
Labour				
Labour force - concrete pouring				
	euro/m3	62.32	4.50	280.42
Labour force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	9347.40	0.17	1589.06
Formwork				
	m^2	263.52	11.00	2898.72
			Total:	**17542.98€**

Table C.5: Cost of columns for the slabs with beams – span 7.20m

Concrete C30/37			
m^3	70.90	77.50	5494.91
Steel A500			
kg	10635.30	0.85	9040.01
Labour			
Labour force - concrete pouring			
euro/m3	70.90	4.50	319.06
Labour force - placing reinforcement (cutting, , bending, installing)			
euro/kg	10635.30	0.17	1808.00
Formwork			
m^2	281.52	11.00	3096.72
		Total:	**19758.69€**

Table C.6: Cost of columns for the slabs with beams – span 6.40m

Concrete C30/37			
m^3	64.87	77.50	5027.58
Steel A500			
kg	9730.80	0.85	8271.18
Labour			
Labour force - concrete pouring			
euro/m3	64.87	4.50	291.92
Labour force - placing reinforcement (cutting, , bending, installing)			
euro/kg	9730.80	0.17	1654.24
Formwork			
m^2	269.28	11.00	2962.08
		Total:	**18207.00€**

Table C.7: Cost of columns for the flat plates – span 5.60m

Concrete C30/37				
	m^3	39.83	77.50	3087.14
Steel A500				
	kg	5975.10	0.78	4660.58
Labour				
Labour force - concrete pouring				
	euro/m3	39.83	4.50	179.25
Labour force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	5975.10	0.17	1015.77
Formwork				
	m^2	210.96	11.00	2320.56
			Total:	**11263.29€**

Table C.8: Cost of columns for the flat plates – span 6.40m

Concrete C30/37				
	m^3	46.86	77.50	3631.88
Steel A500				
	kg	7029.45	0.78	5482.97
Labour				
Labour force - concrete pouring				
	euro/m3	46.86	4.50	210.88
Labour force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	7029.45	0.17	1195.01
Formwork				
	m^2	228.96	11.00	2518.56
			Total:	**13039.30€**

Table C.9: Cost of columns for the flat plates – span 7.20m

Concrete C30/37				
	m^3	57.02	77.50	4419.36
Steel A500				
	kg	8553.60	0.78	6671.81
Labour				
Labour force - concrete pouring				
	euro/m3	57.02	4.50	256.61
Labour force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	8553.60	0.17	1454.11
Formwork				
	m^2	252.00	11.00	2772.00
			Total:	**15573.89€**

Table C.10: Cost of columns for the waffle slabs – span 7.20m

Concrete C30/37				
	m^3	55.78	77.50	4323.11
Steel A500				
	kg	8367.30	0.78	6526.49
Labour				
Labour force - concrete pouring				
	euro/m3	55.78	4.50	251.02
Labour force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	8367.30	0.17	1422.44
Formwork				
	m^2	249.84	11.00	2748.24
			Total:	**15271.30€**

Table C.11: Cost of columns for the waffle slabs – span 8.00m

Concrete C30/37				
	m^3	68.58	77.50	5314.95
Steel A500				
	kg	10287.00	0.78	8023.86
Labour				
Labour force - concrete pouring				
	euro/m3	68.58	4.50	308.61
Labour force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	10287.00	0.17	1748.79
Formwork				
	m^2	276.48	11.00	3041.28
			Total:	**18437.49€**

Table C.12: Cost of columns for the waffle slabs – span 8.80m

Concrete C30/37				
	m^3	76.62	77.50	5937.82
Steel A500				
	kg	11492.55	0.78	8964.19
Labour				
Labour force - concrete pouring				
	euro/m3	76.62	4.50	344.78
Labour force - placing reinforcement (cutting, , bending, installing)				
	euro/kg	11492.55	0.17	1953.73
Formwork				
	m^2	291.60	11.00	3207.60
			Total:	**20408.12€**